PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 8 : Remainder and Factor Theorems

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used

Remainder Theorem: when f(x) is divided by (x − a), the remainder is f(a). For a divisor (px − q), put px − q = 0 and use $f(\frac{q}{p})$.

Factor Theorem: (x − a) is a factor of f(x) ⇔ f(a) = 0. To factorise: find a root by trial (factors of the constant term), divide, then factorise the quotient.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(a)** 2	f(2) = 8 − 8 + 6 − 4 = 2.
(ii)	**(b)** 5	f(3) = 9 − 3k + 6 = 0 ⇒ k = 5.
(iii)	**(a)**	x = −½: 2(−⅛) − (−½) + 5 = −¼ + ½ + 5 = $\frac{21}{4}$.
(iv)	**(c)**	f(−2) = −8 + 8 = 0, so (x + 2) is a factor: (A) is true. (R) is the Factor Theorem, which explains (A).
(v)	**(c)** 3	f(1) = 1 − 6 + a + 4 = a − 1 = 2 ⇒ a = 3.
(vi)	**(d)**	f(3) = 27 − 27 − 3 + 3 = 0, so (x − 3) is a factor. (f(−3) = −48, f(2) = −3, f(−2) = −15 are non-zero.)
(vii)	**(b)** −5	f(1) = 1 + 1 + 3 = 5. Adding k: 5 + k = 0 ⇒ k = −5.
(viii)	**(b)**	For (2x − 3), the remainder is $f(\frac{3}{2})$, not f(3); (A) is false. (R) is the correct statement of the theorem.

Question 2 [4 + 4 + 4 = 12]

(i)

(a) x − 2 = 0 ⇒ x = 2. Remainder = f(2) = 3(8) − 4(4) + 2 − 5 = 24 − 16 + 2 − 5 [1]

Remainder = 5 [1]

(b) 2x + 1 = 0 ⇒ $x = -\frac{1}{2}$. Remainder = g(−½) = 4(−⅛) − 2(¼) + (−½) − 3 = −½ − ½ − ½ − 3 [1]

Remainder = $-\frac{9}{2}$ [1]

(ii)

Divisor x − 1 ⇒ x = 1.

First remainder = a(1) + 3(1) − 2(1) + 4 = a + 5 [1]

Second remainder = 2(1) − a(1) + 3(1) − 4 = 1 − a [1]

Equal remainders: a + 5 = 1 − a [1]

$2a = -4 \Rightarrow \mathbf{a = -2}$ (both remainders = 3 ✓) [1]

(iii)

Let $f(x) = 2x^3 + 3x^2 - 11x - 6$. $x + 3 = 0 \Rightarrow x = -3$ [1]

$f(-3) = 2(-27) + 3(9) - 11(-3) - 6 = -54 + 27 + 33 - 6 = 0$, so $(x + 3)$ is a factor [1]

Dividing: $2x^3 + 3x^2 - 11x - 6 = (x + 3)(2x^2 - 3x - 2)$ [1]

$2x^2 - 3x - 2 = (2x + 1)(x - 2)$. **$f(x) = (x + 3)(x - 2)(2x + 1)$** [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]

(i)

$2x + 1 = 0 \Rightarrow x = -\frac{1}{2}$. $(2x + 1)$ is a factor $\Rightarrow f(-½) = 0$ [1]

$4(-⅛) + k(¼) - 3(-½) + 2 = 0 \Rightarrow -½ + \frac{k}{4} + \frac{3}{2} + 2 = 0$ [1]

$\frac{k}{4} = -3 \Rightarrow \mathbf{k = -12}$ [1]

(ii)

$(x - 2)$ is a factor: $f(2) = 16 + 4a + 2b - 6 = 0 \Rightarrow 2a + b = -5$ …(1) [1]

Remainder −12 for $(x + 1)$: $f(-1) = -2 + a - b - 6 = -12 \Rightarrow a - b = -4$ …(2) [1]

Adding (1) and (2): $3a = -9 \Rightarrow \mathbf{a = -3}$; from (2), **b = 1** [1]

(iii)

Divisor $x + 1 \Rightarrow x = -1$.

Remainder of $px^3 + 4x^2 - 3$: $-p + 4 - 3 = 1 - p$ [1]

Remainder of $2x^3 - 5x + p$: $-2 + 5 + p = 3 + p$ [1]

$1 - p = 3 + p \Rightarrow 2p = -2 \Rightarrow \mathbf{p = -1}$ [1]

Common remainder = $1 - (-1) =$ **2** (check: $3 + (-1) = 2$ ✓) [1]

Question 4 [3 + 3 + 4 = 10]

(i)

$f(2) = 8 - 16 + 2 + 6 = 0$, so $(x - 2)$ is a factor [1]

$x^3 - 4x^2 + x + 6 = (x - 2)(x^2 - 2x - 3)$ [1]

$x^2 - 2x - 3 = (x - 3)(x + 1)$. **$= (x - 2)(x - 3)(x + 1)$** [1]

(ii)

Try $x = -2$: $f(-2) = 2(-8) - 4 - 13(-2) - 6 = -16 - 4 + 26 - 6 = 0 \Rightarrow (x + 2)$ is a factor [1]

$2x^3 - x^2 - 13x - 6 = (x + 2)(2x^2 - 5x - 3)$ [1]

$2x^2 - 5x - 3 = (2x + 1)(x - 3)$. **$= (x + 2)(2x + 1)(x - 3)$** [1]

(iii)

$f(2) = 24 + 4 - 40 + 12 = 0 \Rightarrow (x - 2)$ is a factor [1]

$3x^3 + x^2 - 20x + 12 = (x - 2)(3x^2 + 7x - 6)$ [1]

$3x^2 + 7x - 6 = 3x^2 + 9x - 2x - 6 = (x + 3)(3x - 2)$. **$= (x - 2)(x + 3)(3x - 2)$** [1]

Solving $(x - 2)(x + 3)(3x - 2) = 0$: **$x = 2, x = -3, x = \frac{2}{3}$** [1]

Question 5 [3 + 3 + 4 = 10]

(i)

Let $f(x) = x^3 + 2x^2 - x - 2$.

$f(1) = 1 + 2 - 1 - 2 = 0 \Rightarrow (x - 1)$ is a factor [1]

$f(-3) = -27 + 18 + 3 - 2 = -8 \neq 0 \Rightarrow (x + 3)$ is not a factor [1]

$f(-2) = -8 + 8 + 2 - 2 = 0 \Rightarrow (x + 2)$ is a factor. **$(x - 1)$ and $(x + 2)$ are factors** [1]

(ii)

Let the number added be k. The polynomial is $3x^3 - x^2 + 2x + k$; $x + 1 = 0 \Rightarrow x = -1$.

Remainder $= 3(-1) - (1) + 2(-1) + k = -6 + k$ [1]

$-6 + k = -2$ [1]

$k = 4$ (the number to be added is 4) [1]

(iii)

$(x + 2)$ is a factor: $f(-2) = -16 + 4a + 14 + b = 0 \Rightarrow 4a + b = 2$ …(1) [1]

$(x - 3)$ is a factor: $f(3) = 54 + 9a - 21 + b = 0 \Rightarrow 9a + b = -33$ …(2) [1]

Subtracting: $5a = -35 \Rightarrow$ **$a = -7$**; from (1), **$b = 30$** [1]

$f(x) = 2x^3 - 7x^2 - 7x + 30 = (x + 2)(x - 3)(2x - 5)$, since $(x + 2)(x - 3) = x^2 - x - 6$ and $2x^3 - 7x^2 - 7x + 30 = (x^2 - x - 6)(2x - 5)$. **$f(x) = (x + 2)(x - 3)(2x - 5)$** [1]

Question 6 [3 + 3 + 4 = 10]

(i)

Remainder for $(x - 2)$: $f(2) = 8 + 4k - 6 + 4 = 4k + 6$ [1]

Remainder for $(x + 1)$: $f(-1) = -1 + k + 3 + 4 = k + 6$ [1]

$4k + 6 = 2(k + 6) \Rightarrow 2k = 6 \Rightarrow$ **$k = 3$** (remainders 18 and 9 ✓) [1]

(ii)

$f(1) = 1 - 13 + 12 = 0 \Rightarrow (x - 1)$ is a factor [1]

$x^3 - 13x + 12 = (x - 1)(x^2 + x - 12)$ [1]

$x^2 + x - 12 = (x + 4)(x - 3)$. **$= (x - 1)(x - 3)(x + 4)$** [1]

(iii)

$(x + 1)$ is a factor: $f(-1) = -2 + a - b + 3 = 0 \Rightarrow a - b = -1$ …(1) [1]

$(2x - 3)$ is a factor: $f(\frac{3}{2}) = \frac{27}{4} + \frac{9a}{4} + \frac{3b}{2} + 3 = 0 \Rightarrow 3a + 2b = -13$ …(2) [1]

From (1), $a = b - 1$. Put in (2): $3b - 3 + 2b = -13 \Rightarrow 5b = -10 \Rightarrow$ **$b = -2$, $a = -3$** [1]

$f(x) = 2x^3 - 3x^2 - 2x + 3 = (x + 1)(2x^2 - 5x + 3) =$ **$(x + 1)(x - 1)(2x - 3)$** [1]

— End of Solutions —