PRACTICE PAPER

ICSE CLASS 10 – MATHEMATICS

Chapter 9 : Matrices

STEP-BY-STEP SOLUTIONS & MARKING SCHEME · Maximum Marks: 40

Results used

Equal matrices: same order and equal corresponding elements. Addition/subtraction: only for matrices of the same order, element by element. Scalar multiple kA: multiply every element by k.

Product AB exists only if (columns of A) = (rows of B); if A is m × n and B is n × p then AB is m × p. Entry (i, j) of AB = (row i of A) × (column j of B). In general AB ≠ BA.

SECTION A (20 Marks)

Question 1 [8 × 1 = 8]

Part	Ans.	Working
(i)	**(d)**	Order m × n needs m × n = 12. 5 × 2 = 10, so it is not possible.
(ii)	**(b)** 10	x + 1 = 4 ⇒ x = 3; y − 2 = 5 ⇒ y = 7; x + y = 10.
(iii)	**(a)** 2	$A^t = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$, so the element in the 2nd row, 1st column is 2.
(iv)	**(b)**	A has 3 columns but B has 2 rows, so AB is not possible: (A) is false; (R) is true.
(v)	**(a)**	(1 × 2)(2 × 1) gives a 1 × 1 matrix: [2 × 4 + 3 × 5] = [23].
(vi)	**(a)**	$A^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1+0 & 2+2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$.
(vii)	**(a)**	$X = ½ \times \begin{pmatrix} 4 & -6 \\ 2 & 8 \end{pmatrix} = \begin{pmatrix} 2 & -3 \\ 1 & 4 \end{pmatrix}$.
(viii)	**(b)**	Matrix multiplication is not commutative, so (A) is false; (R) is true.

Question 2 [4 + 4 + 4 = 12]

(i)

Equal matrices ⇒ corresponding elements are equal: x + 2y = 7 …(1), 2x − y = 4 …(2), z − 1 = 4 …(3) [1]

From (2): y = 2x − 4. Put in (1): x + 4x − 8 = 7 ⇒ 5x = 15 ⇒ **x = 3** [1]

y = 2(3) − 4 = **2** [1]

From (3): **z = 5** [1]

(ii)

$2A = \begin{pmatrix} 4 & -2 \\ 6 & 0 \end{pmatrix}$ [1]

$3B = \begin{pmatrix} 3 & 12 \\ -6 & 15 \end{pmatrix}$ [1]

$$2A - 3B = \begin{pmatrix} 4-3 & -2-12 \\ 6+6 & 0-15 \end{pmatrix}$$ [1]

$$\mathbf{2A - 3B = \begin{pmatrix} 1 & -14 \\ 12 & -15 \end{pmatrix}}$$ [1]

(iii)

$$AB = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 1\times2+3\times1 & 1\times0+3\times4 \\ 2\times2+(-1)\times1 & 2\times0+(-1)\times4 \end{pmatrix}$$ [1]

$$\mathbf{AB = \begin{pmatrix} 5 & 12 \\ 3 & -4 \end{pmatrix}}$$ [1]

$$BA = \begin{pmatrix} 2 & 0 \\ 1 & 4 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 2\times1+0\times2 & 2\times3+0\times(-1) \\ 1\times1+4\times2 & 1\times3+4\times(-1) \end{pmatrix} = \mathbf{\begin{pmatrix} 2 & 6 \\ 9 & -1 \end{pmatrix}}$$ [1]

The two products are different, so **AB ≠ BA** [1]

SECTION B (20 Marks) — any two questions

Question 3 [3 + 3 + 4 = 10]

(i)

$$\begin{pmatrix} 2x \\ x \end{pmatrix} + \begin{pmatrix} -y \\ 3y \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix} \Rightarrow \begin{pmatrix} 2x-y \\ x+3y \end{pmatrix} = \begin{pmatrix} 4 \\ 9 \end{pmatrix}$$ [1]

$2x - y = 4$ …(1) $x + 3y = 9$ …(2). From (1), $y = 2x - 4$; put in (2): $x + 6x - 12 = 9 \Rightarrow 7x = 21$ [1]

x = 3, y = 2 (check: $6 - 2 = 4$; $3 + 6 = 9$ ✓) [1]

(ii)

$$3X = \begin{pmatrix} 5 & 2 \\ 6 & 13 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 0 & 4 \end{pmatrix}$$ [1]

$$3X = \begin{pmatrix} 3 & 3 \\ 6 & 9 \end{pmatrix}$$ [1]

$$X = \tfrac{1}{3} \times \begin{pmatrix} 3 & 3 \\ 6 & 9 \end{pmatrix} = \mathbf{\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}}$$ [1]

(iii)

$$A^2 = \begin{pmatrix} 3 & 1 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ -2 & 0 \end{pmatrix} = \begin{pmatrix} 9-2 & 3+0 \\ -6+0 & -2+0 \end{pmatrix} = \begin{pmatrix} 7 & 3 \\ -6 & -2 \end{pmatrix}$$ [1]

$$3A = \begin{pmatrix} 9 & 3 \\ -6 & 0 \end{pmatrix}, \quad 2I = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$ [1]

$$A^2 - 3A + 2I = \begin{pmatrix} 7-9+2 & 3-3+0 \\ -6+6+0 & -2-0+2 \end{pmatrix}$$ [1]

$$= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O.$$ **Hence proved.** [1]

Question 4 [3 + 3 + 4 = 10]

(i)

$$\begin{pmatrix} 2x+y \\ 3x-2y \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \Rightarrow 2x + y = 5 \ \ldots(1), \quad 3x - 2y = 4 \ \ldots(2)$$ [1]

(1) × 2 + (2): $4x + 2y + 3x - 2y = 10 + 4 \Rightarrow 7x = 14 \Rightarrow$ **x = 2** [1]

From (1): y = 5 − 4 = **1** (check: 6 − 2 = 4 ✓) [1]

(ii)

Let M be of order a × b. The first matrix on the right is 2 × 2, so b = 2; the result is 1 × 2, so a = 1. **Order of M = 1 × 2** [1]

Let M = [x y]. Then [2x + 3y x − y] = [12 1] ⇒ 2x + 3y = 12, x − y = 1 [1]

x = y + 1 ⇒ 2y + 2 + 3y = 12 ⇒ y = 2, x = 3. **M = [3 2]** [1]

(iii)

$$AB = \begin{pmatrix} 2\times2 + 3\times(-1) & 2\times(-3) + 3\times2 \\ 1\times2 + 2\times(-1) & 1\times(-3) + 2\times2 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$ [1]

$$BA = \begin{pmatrix} 2\times2 + (-3)\times1 & 2\times3 + (-3)\times2 \\ (-1)\times2 + 2\times1 & (-1)\times3 + 2\times2 \end{pmatrix} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$$ [1]

$$A^2 = \begin{pmatrix} 2\times2 + 3\times1 & 2\times3 + 3\times2 \\ 1\times2 + 2\times1 & 1\times3 + 2\times2 \end{pmatrix} = \begin{pmatrix} \mathbf{7} & \mathbf{12} \\ \mathbf{4} & \mathbf{7} \end{pmatrix}$$ [1]

AB = BA = I: here the two products are equal (each is the unit matrix), so B behaves as the multiplicative inverse of A. [1]

Question 5 [3 + 3 + 4 = 10]

(i)

$$LHS = \begin{pmatrix} x+1 & 0 \\ 6 & y+3 \end{pmatrix};\quad RHS = \begin{pmatrix} 6 & 0 \\ 6 & 6 \end{pmatrix}$$ [1]

x + 1 = 6 ⇒ **x = 5** [1]

y + 3 = 6 ⇒ **y = 3** (the other elements 0 = 0 and 6 = 6 agree ✓) [1]

(ii)

$$A^2 = \begin{pmatrix} 2 & x \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 2x + x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 3x \\ 0 & 1 \end{pmatrix}$$ [1]

$$A^2 = B \Rightarrow \begin{pmatrix} 4 & 3x \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 15 \\ 0 & 1 \end{pmatrix}$$ [1]

3x = 15 ⇒ **x = 5** [1]

(iii)

$$A^2 = \begin{pmatrix} 4\times4 + 1\times2 & 4\times1 + 1\times3 \\ 2\times4 + 3\times2 & 2\times1 + 3\times3 \end{pmatrix} = \begin{pmatrix} 18 & 7 \\ 14 & 11 \end{pmatrix}$$ [1]

$$AB = \begin{pmatrix} 4\times1 + 1\times(-2) & 4\times0 + 1\times1 \\ 2\times1 + 3\times(-2) & 2\times0 + 3\times1 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ -4 & 3 \end{pmatrix}$$ [1]

$$2C = \begin{pmatrix} 4 & -2 \\ 0 & 6 \end{pmatrix}$$ [1]

$$A^2 - AB + 2C = \begin{pmatrix} 18 - 2 + 4 & 7 - 1 - 2 \\ 14 + 4 + 0 & 11 - 3 + 6 \end{pmatrix} = \begin{pmatrix} \mathbf{20} & \mathbf{4} \\ \mathbf{18} & \mathbf{14} \end{pmatrix}$$ [1]

Question 6 [3 + 3 + 4 = 10]

(i)

The left matrix is 2 × 2 and the result is 2 × 1, so P is of order 2 × 1. **Order of P = 2 × 1** [1]

Let $P = \begin{pmatrix} a \\ b \end{pmatrix}$. Then $\begin{pmatrix} 3a + b \\ 2a + b \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$ ⇒ 3a + b = 5, 2a + b = 3 [1]

Subtracting: a = 2; then b = 3 − 4 = −1. $\mathbf{P} = \begin{pmatrix} \mathbf{2} \\ \mathbf{-1} \end{pmatrix}$ [1]

(ii)

(a) **False**: matrix multiplication is not commutative (e.g. in Q2(iii), AB ≠ BA) [1]

(b) **True**: matrix multiplication is associative [1]

(c) **False**: $(A + B)^2 = A^2 + AB + BA + B^2$, and AB ≠ BA in general, so it cannot be written as $A^2 + 2AB + B^2$ [1]

(iii)

2X + Y = P …(1) X − Y = Q …(2), where P and Q are the given matrices [1]

Adding (1) and (2): $3X = \begin{pmatrix} 6 & 3 \\ 6 & 3 \end{pmatrix} \Rightarrow \mathbf{X} = \begin{pmatrix} \mathbf{2} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} \end{pmatrix}$ [1]

From (2): $Y = X - Q = \begin{pmatrix} 2-1 & 1+1 \\ 2-4 & 1-2 \end{pmatrix}$ [1]

$\mathbf{Y} = \begin{pmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{-2} & \mathbf{-1} \end{pmatrix}$ (check: $2X + Y = \begin{pmatrix} 5 & 4 \\ 2 & 1 \end{pmatrix}$ ✓) [1]

— End of Solutions —