CLASS XI – MATHEMATICS (041)

CHAPTER 10 : CONIC SECTIONS

PRACTICE PAPER – STEP-BY-STEP SOLUTIONS & MARKING SCHEME

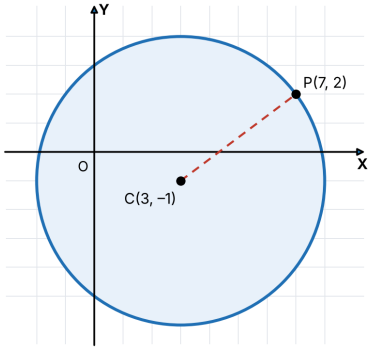
Section A – Answer Key (1 mark each)

Q1	Q2	Q3	Q4	Q5	Q6	Q7
C	B	D	A	C	D	B
Q8	**Q9**	**Q10**	**Q11**	**Q12**	**Q13**	**Q14**
C	A	D	A	B	D	A

SECTION – A (14 × 1 = 14)

Multiple Choice and Assertion–Reason Questions

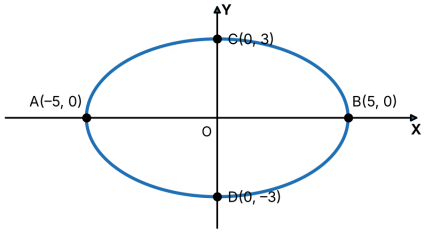
Q1. Answer: (C) — 1 mark



Radius r = CP = $\sqrt{((7-3)^2 + (2+1)^2)} = \sqrt{(16+9)} = 5$.

Equation $(x-h)^2 + (y-k)^2 = r^2$ with (h, k) = (3, –1): **$(x-3)^2 + (y+1)^2 = 25$**.

Q2. Answer: (B) — 1 mark



From the figure the major axis is along the x-axis with a = 5 and b = 3.

$c = \sqrt{(a^2 - b^2)} = \sqrt{(25-9)} = 4$, so $e = c/a =$ **4/5**.

Q3. Answer: (D) — 1 mark

Complete the squares: $(x^2 - 6x + 9) + (y^2 + 4y + 4) = 3 + 9 + 4 \Rightarrow (x-3)^2 + (y+2)^2 = 16$.

Centre **(3, –2)**, radius **4**.

Q4. Answer: (A) — 1 mark

$x^2 = -12y$ is of the form $x^2 = -4ay$ with $4a = 12$, i.e. $a = 3$. The parabola opens downwards.

Focus = (0, –a) = **(0, –3)** and directrix y = a, i.e. **y = 3**.

Q5. Answer: (C) — 1 mark

$16x^2 + 25y^2 = 400 \Rightarrow x^2/25 + y^2/16 = 1$, so a = 5, b = 4.

Latus rectum $= 2b^2/a = 2(16)/5 =$ **32/5**.

Q6. Answer: (D) — 1 mark

$a^2 = 144$, $b^2 = 25 \Rightarrow c^2 = a^2 + b^2 = 169 \Rightarrow c = 13$.

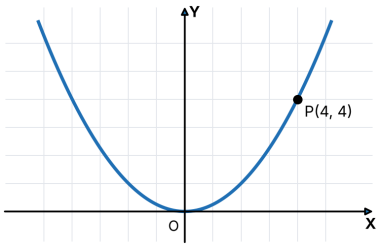
$e = c/a =$ **13/12**.

Q7. Answer: (B) — 1 mark

Vertices and foci lie on the y-axis, so the form is $x^2/b^2 + y^2/a^2 = 1$ with a = 10, c = 6.

$b^2 = a^2 - c^2 = 100 - 36 = 64$. The equation is **$x^2/64 + y^2/100 = 1$**.

Q8. Answer: (C) **1 mark**



The axis is the y-axis and the parabola opens upwards, so $x^2 = 4ay$. Through (4, 4): $16 = 4a(4) \Rightarrow a = 1$. The focus is (0, a) = **(0, 1)**.

Q9. Answer: (A) **1 mark**

A plane cutting both nappes ($0 \le \beta < \alpha$) gives a **hyperbola**. ($\beta = \alpha$ gives a parabola, $\alpha < \beta < 90°$ an ellipse and $\beta = 90°$ a circle.)

Q10. Answer: (D) **1 mark**

Centre = mid-point of AB = $((-1 + 5)/2, (2 - 6)/2) = (2, -2)$.
Radius = AB/2 = $\sqrt{(6^2 + 8^2)}/2 = 10/2 = 5$. The equation is **$(x - 2)^2 + (y + 2)^2 = 25$**.

Q11. Answer: (A) **1 mark**

$(-6)^2 = 4a(3) \Rightarrow 36 = 12a \Rightarrow a = 3$.
Length of latus rectum = $4a$ = **12**.

Q12. Answer: (B) **1 mark**

$9x^2 - 16y^2 = 144 \Rightarrow x^2/16 - y^2/9 = 1$, so $a = 4$.
Length of the transverse axis = $2a$ = **8**.

Q13. Answer: (D) **1 mark**

Since $c^2 = a^2 - b^2 < a^2$, we get $0 < e = c/a < 1$. So the Reason is true.
The eccentricity of an ellipse is always less than 1, so the Assertion is false → option (D).

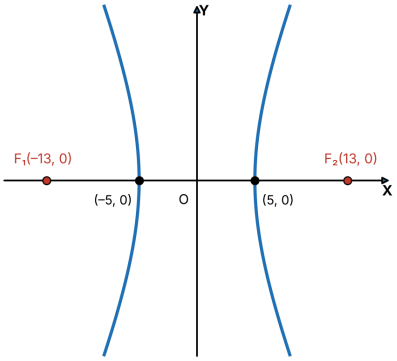
Q14. Answer: (A) **1 mark**

$y^2 = -8x$ is of the form $y^2 = -4ax$ with $a = 2$; the coefficient of x is negative, so the curve opens to the left. Assertion is true.
The Reason states exactly this rule, so it explains the Assertion → option (A).

SECTION – B (4 × 2 = 8)

Very Short Answer Questions

Q15. 2 marks



(i) $a = 5$, $c = 13 \Rightarrow b^2 = c^2 - a^2 = 169 - 25 = 144$. The equation is $\mathbf{\frac{x^2}{25} - \frac{y^2}{144} = 1}$. **[1]**

(ii) $e = c/a = \mathbf{13/5}$. Latus rectum $= 2b^2/a = 2(144)/5 = \mathbf{288/5}$. **[1]**

Q16. 2 marks

$(x^2 + 6x + 9) + (y^2 - 8y + 16) = 11 + 9 + 16 \Rightarrow (x + 3)^2 + (y - 4)^2 = 36$. **[1]**

Centre = **(–3, 4)** and radius = **6**. **[1]**

Q17. 2 marks

(A) $x^2 = 20y$ is of the form $x^2 = 4ay$ with $a = 5$; the axis is the y-axis and the parabola opens upwards. **[½]**

Focus = **(0, 5)**, directrix: **y = –5**, latus rectum $= 4a =$ **20**. **[1½]**

OR (B) The point (4, –2) lies in the fourth quadrant, so the parabola opens downwards: $x^2 = -4ay$. **[1]**

$16 = -4a(-2) = 8a \Rightarrow a = 2$. The equation is $\mathbf{x^2 = -8y}$. **[1]**

Q18. 2 marks

(A) Foci on the x-axis: $x^2/a^2 + y^2/b^2 = 1$ with $c = 3$ and $2b = 8 \Rightarrow b = 4$. **[1]**

$a^2 = b^2 + c^2 = 16 + 9 = 25$. The equation is $\mathbf{\frac{x^2}{25} + \frac{y^2}{16} = 1}$. **[1]**

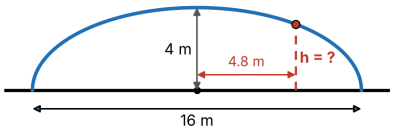
OR (B) Foci on the y-axis: $y^2/a^2 - x^2/b^2 = 1$ with $a = 4$, $c = 6$. **[1]**

$b^2 = c^2 - a^2 = 36 - 16 = 20$. The equation is $\mathbf{\frac{y^2}{16} - \frac{x^2}{20} = 1}$. **[1]**

SECTION – C (3 × 3 = 9)

Short Answer Questions

Q19. 3 marks



(i) The semi-axes are a = 16/2 = 8 and b = 4. The equation is $\mathbf{x^2/64 + y^2/16 = 1}$. **[1]**
(ii) Put x = 4.8: $y^2 = 16(1 - 4.8^2/64) = 16(1 - 0.36) = 16 \times 0.64 = 10.24 \Rightarrow h = \mathbf{3.2\ m}$. **[1]**
(iii) Put $y = 2\sqrt{3}$: $12/16 + x^2/64 = 1 \Rightarrow x^2 = 64 \times 1/4 = 16 \Rightarrow x = \mathbf{4\ m}$ from the centre. **[1]**

Q20. 3 marks

(A) Let the ellipse be $x^2/a^2 + y^2/b^2 = 1$. It passes through (0, 2): $4/b^2 = 1 \Rightarrow b^2 = 4$. **[1]**
It passes through $(2, \sqrt{3})$: $4/a^2 + 3/4 = 1 \Rightarrow 4/a^2 = 1/4 \Rightarrow a^2 = 16$. **[1]**
The equation is $\mathbf{x^2/16 + y^2/4 = 1}$ ($a^2 > b^2$, so the major axis is along the x-axis). **[1]**
OR (B) $4x^2 + y^2 = 36 \Rightarrow x^2/9 + y^2/36 = 1$. The larger denominator is under y^2, so the major axis is along the y-axis: $a^2 = 36$, $b^2 = 9$. **[1]**
$c = \sqrt{(36 - 9)} = 3\sqrt{3}$. Foci: $\mathbf{(0, \pm 3\sqrt{3})}$; $e = c/a = \mathbf{\sqrt{3}/2}$. **[1]**
Latus rectum $= 2b^2/a = 2(9)/6 = \mathbf{3}$. **[1]**

Q21. 3 marks

(A) The centre (h, k) lies on x – y = 0, so k = h. Let the centre be (h, h). **[1]**
Equal radii: $(4 - h)^2 + h^2 = h^2 + (2 - h)^2 \Rightarrow 16 - 8h = 4 - 4h \Rightarrow h = 3$. Centre (3, 3). **[1]**
$r^2 = (4 - 3)^2 + 3^2 = 10$. The equation is $\mathbf{(x - 3)^2 + (y - 3)^2 = 10}$. **[1]**
OR (B) Foci on the x-axis: $x^2/a^2 - y^2/b^2 = 1$ with c = 3. Latus rectum $2b^2/a = 16 \Rightarrow b^2 = 8a$. **[1]**
$c^2 = a^2 + b^2 \Rightarrow 9 = a^2 + 8a \Rightarrow a^2 + 8a - 9 = 0 \Rightarrow (a + 9)(a - 1) = 0 \Rightarrow a = 1$ ($a > 0$), $b^2 = 8$. **[1]**
The equation is $\mathbf{x^2 - y^2/8 = 1}$, i.e. $8x^2 - y^2 = 8$. **[1]**

SECTION – D (1 × 5 = 5)

Long Answer Question

Q22. **5 marks**

(A) AB = 10 cm and AP = 4 cm, so PB = 6 cm. Let the rod make an angle θ with OX. From P draw PQ perpendicular to OY and PR perpendicular to OX, so PQ = x and PR = y. **[1]**

In ΔPBQ: angle BPQ = θ (PQ is parallel to OX), so cos θ = PQ/PB = x/6. **[1½]**

In ΔPRA: sin θ = PR/PA = y/4. **[1½]**

Since $\cos^2\theta + \sin^2\theta = 1$: $(x/6)^2 + (y/4)^2 = 1$, i.e. $\mathbf{x^2/36 + y^2/16 = 1}$. **[½]**

This is the equation of an ellipse (semi-axes 6 and 4), so the locus of P is an ellipse. **[½]**

OR (B)(i) Let the parabola be $x^2 = -4ay$ (vertex at the crown, opening downwards). The end of the arch is 12 m sideways and 9 m below the crown: (12, –9). **[1]**

$144 = -4a(-9) = 36a \Rightarrow a = 4$. **[1]**

The equation is $\mathbf{x^2 = -16y}$. **[1]**

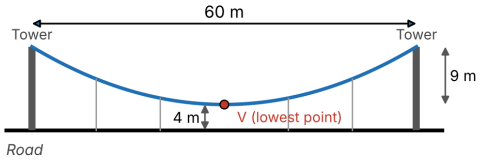
(ii) At 4 m below the crown, $y = -4$: $x^2 = -16(-4) = 64 \Rightarrow x = \pm 8$. **[1]**

Width = 2 × 8 = **16 m**. **[1]**

SECTION – E (1 × 4 = 4)

Case Study Based Question

Q23. **4 marks**



(i) The vertex is at the origin and the axis is the y-axis, so $x^2 = 4ay$. The tower top is (30, 9): $900 = 36a \Rightarrow a = 25$. The equation is $\mathbf{x^2 = 100y}$. **[1]**

(ii) At $x = 20$: $y = 400/100 = 4$ m above V. V is 4 m above the road, so hanger length = 4 + 4 = **8 m**. **[1]**

(iii)(a) Put $y = 6.25$: $x^2 = 100 \times 6.25 = 625$. **[1]**

$x =$ **25 m** from the middle of the bridge. **[1]**

OR (b) Here $4a = 100$, $a = 25$. The focus is **(0, 25)** and the directrix is **y = –25**. **[1]**

Length of latus rectum $= 4a =$ **100 m**. **[1]**

— End of Solutions —