CLASS XI – MATHEMATICS (041)

CHAPTER 12 : LIMITS AND DERIVATIVES

PRACTICE PAPER – STEP-BY-STEP SOLUTIONS & MARKING SCHEME

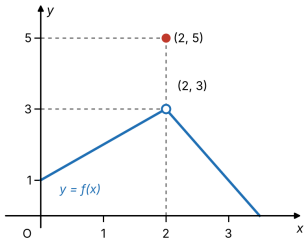
Section A – Answer Key (1 mark each)

Q1	Q2	Q3	Q4	Q5	Q6	Q7
C	D	A	B	C	A	D
Q8	**Q9**	**Q10**	**Q11**	**Q12**	**Q13**	**Q14**
B	A	D	B	C	C	B

SECTION – A (14 × 1 = 14)

Multiple Choice and Assertion–Reason Questions

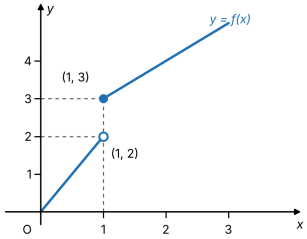
Q1. Answer: (C) 1 mark



Left of 2, f(x) = x + 1 → 3. Right of 2, f(x) = 7 – 2x → 3. Both approach the open circle (2, 3), so $\lim_{x\to 2} f(x) = 3$.

The dot at (2, 5) gives f(2) = 5. Hence the required value is 3 + 5 = **8**.

Q2. Answer: (D) 1 mark



Left hand limit at 1 = 2 (open circle), right hand limit at 1 = 3 (dot at (1, 3)).

The two limits are different, so $\lim_{x\to 1} f(x)$ **does not exist**, even though f(1) = 3 is defined.

Q3. Answer: (A) 1 mark

Using $\lim_{x\to a} (x^n - a^n)/(x - a) = na^{n-1}$ with n = 3, a = 2: $3 \times 2^2 =$ **12**.

Q4. Answer: (B) 1 mark

tan 5x / x = (sin 5x / 5x) × 5 × (1 / cos 5x).

As x → 0: sin 5x / 5x → 1 and cos 5x → 1, so the limit = 1 × 5 × 1 = **5**.

Q5. Answer: (C) 1 mark

$f'(x) = 4x^3 - 6x$, so $f'(2) = 4(8) - 6(2) = 32 - 12 =$ **20**.

Q6. Answer: (A) 1 mark

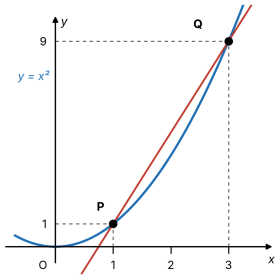
Left hand limit at 3 = 2(3) + k = 6 + k. Right hand limit at 3 = 10.

The limit exists only if the two are equal: 6 + k = 10 ⇒ k = **4**.

Q7. Answer: (D) 1 mark

Quotient rule with u = x, v = x + 2: $(u'v - uv')/v^2 = [1(x + 2) - x(1)]/(x + 2)^2 =$ **$2/(x + 2)^2$**.

Q8. Answer: (B) **1 mark**



Slope of chord PQ = (9 – 1)/(3 – 1) = 8/2 = 4.
Slope of the tangent at P = f′(1) = 2x at x = 1 = 2. So the answer is **4 and 2**.

Q9. Answer: (A) **1 mark**

By the product rule, d/dx ($x^2 \sin x$) = $2x \sin x + x^2 \cos x$.
At x = π/2: $2(\pi/2)(1) + (\pi^2/4)(0) = \boldsymbol{\pi}$.

Q10. Answer: (D) **1 mark**

$1 - \cos 4x = 2 \sin^2 2x$, so the expression = $2 (\sin 2x)^2/x^2 = 2 \times 4 \times (\sin 2x / 2x)^2$.
As x → 0, sin 2x / 2x → 1, so the limit = 2 × 4 × 1 = **8**.

Q11. Answer: (B) **1 mark**

This is the definition of f′(2) for $f(x) = x^5$: $f'(2) = 5 \times 2^4 = \mathbf{80}$.

Q12. Answer: (C) **1 mark**

Put y = π/2 – x, so y → 0 as x → π/2 and cos x = cos(π/2 – y) = sin y.
The limit = $\lim_{y \to 0} (\sin y)/y = \mathbf{1}$.

Q13. Answer: (C) **1 mark**

Assertion: $(x^2 - 9)/(x - 3) = x + 3$ for x ≠ 3, so the limit is 3 + 3 = 6. **(A) is true.**
Reason: the quotient rule for limits needs the limit of the denominator to be non-zero. Here it is 0, so **(R) is false**.
Correct option: **(C)**.

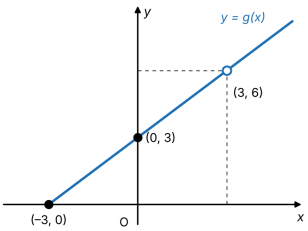
Q14. Answer: (B) **1 mark**

Assertion: f′(x) = 2x, so f′(3) = 6. **(A) is true.**
Reason: the derivative of a constant function is 0, which is a true statement.
But it plays no part in finding the derivative of x^2, so **(R) is not the correct explanation**. Correct option: **(B)**.

SECTION – B (4 × 2 = 8)

Very Short Answer Questions

Q15. 2 marks



(i) At x = 3 the denominator x – 3 = 0, so g(3) is of the form 0/0 and is not defined (open circle). Points on either side of x = 3 approach the point (3, 6), so the graph suggests $\lim_{x\to 3} g(x) = 6$. **[1]**
(ii) For $x \neq 3$: $g(x) = (x – 3)(x + 3)/(x – 3) = x + 3$. Hence $\lim_{x\to 3} g(x) = 3 + 3 =$ **6**, matching the graph. **[1]**

Q16. 2 marks

Put y = 4 + x, so y → 4 as x → 0 and the expression becomes $(\sqrt{y} – 2)/(y – 4)$. **[½]**
$= (y^{1/2} – 4^{1/2})/(y – 4)$. Using $\lim_{y\to a} (y^n – a^n)/(y – a) = na^{n–1}$ with n = 1/2, a = 4: **[½]**
limit $= (1/2) \times 4^{-1/2} = (1/2)(1/2) =$ **1/4**. **[1]**

Q17. 2 marks

(A) At x = 3 the expression is 0/0, so factorise: $(x – 2)(x – 3)/[(x – 3)(x + 3)]$. **[1]**
Cancelling x – 3 (as $x \neq 3$): $\lim_{x\to 3} (x – 2)/(x + 3) = 1/6$. **Answer: 1/6**. **[1]**
OR (B) $\tan 3x / \sin 4x = (\sin 3x / \cos 3x) \times (1 / \sin 4x) = (\sin 3x / 3x) \times (4x / \sin 4x) \times (3/4) \times (1 / \cos 3x)$. **[1]**
As x → 0 each of the first, second and fourth factors → 1, so the limit = **3/4**. **[1]**

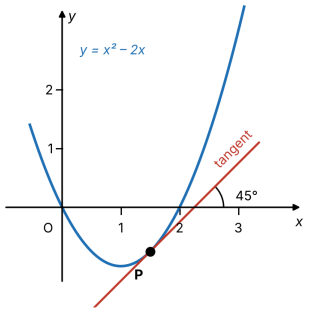
Q18. 2 marks

(A) $f'(x) = \lim_{h\to 0} [f(x + h) – f(x)]/h$ where $f(x + h) = 3(x + h) – (x + h)^2$. **[½]**
$f(x + h) – f(x) = 3h – 2xh – h^2$, so $[f(x + h) – f(x)]/h = 3 – 2x – h$. **[1]**
Taking h → 0: $f'(x) =$ **3 – 2x**. **[½]**
OR (B) Product rule with $u = 2x + 1$, $v = 3x^2 – x$: $f'(x) = 2(3x^2 – x) + (2x + 1)(6x – 1)$. **[1]**
$= 6x^2 – 2x + 12x^2 + 4x – 1 =$ **$18x^2 + 2x – 1$**. **[1]**

SECTION – C (3 × 3 = 9)

Short Answer Questions

Q19. 3 marks



(i) f(x + h) – f(x) = (x + h)² – 2(x + h) – x² + 2x = 2xh + h² – 2h. **[1]**

f′(x) = $\lim_{h\to 0}$ (2xh + h² – 2h)/h = $\lim_{h\to 0}$ (2x + h – 2) = **2x – 2**. **[1]**

(ii) The slope of the tangent at P(a, f(a)) is f′(a) = tan 45° = 1, so 2a – 2 = 1 ⇒ a = 3/2. **[½]**

f(3/2) = 9/4 – 3 = –3/4. Hence **P = (3/2, –3/4)**. **[½]**

Q20. 3 marks

(A) Divide numerator and denominator by x: (3 + sin 2x / x)/(1 + tan x / x). **[1]**

sin 2x / x = 2 × (sin 2x / 2x) → 2 and tan x / x → 1 as x → 0. **[1]**

Limit = (3 + 2)/(1 + 1) = **5/2**. **[1]**

OR (B) Divide numerator and denominator by (x – 1): [(x⁷ – 1)/(x – 1)] ÷ [(√x – 1)/(x – 1)]. **[1]**

Numerator → 7 × 1⁶ = 7; denominator (n = 1/2) → (1/2) × $1^{-1/2}$ = 1/2. **[1]**

Limit = 7 ÷ (1/2) = **14**. **[1]**

Q21. 3 marks

(A) Quotient rule with u = sin x, v = 1 + cos x: u′ = cos x, v′ = –sin x. **[1]**

f′(x) = [cos x(1 + cos x) – sin x(–sin x)]/(1 + cos x)² = [cos x + cos² x + sin² x]/(1 + cos x)². **[1]**

= (cos x + 1)/(1 + cos x)² = **1/(1 + cos x)**. **[1]**

OR (B) f(x + h) – f(x) = 1/(x + h + 2) – 1/(x + 2) = [(x + 2) – (x + h + 2)]/[(x + h + 2)(x + 2)] = –h/[(x + h + 2)(x + 2)]. **[1½]**

Dividing by h: –1/[(x + h + 2)(x + 2)]. As h → 0, this gives f′(x) = **–1/(x + 2)²**. **[1½]**

SECTION – D (1 × 5 = 5)

Long Answer Question

Q22. **5 marks**

(A) Left hand limit at 1 = $\lim_{x \to 1^-} (ax + b) = a + b$. **[1]**

Right hand limit at 1 = $\lim_{x \to 1^+} (2ax - b) = 2a - b$. **[1]**

Given limit = f(1) = 6, so $a + b = 6$ and $2a - b = 6$. **[1]**

Adding: $3a = 12 \Rightarrow a = 4$, and then $b = 2$. **[1]**

For $x > 1$, $f(x) = 8x - 2$, so $\lim_{x \to 2} f(x) = 16 - 2 =$ **14**. **[1]**

OR (B) $f(x + h) - f(x) = \cos(2x + 2h) - \cos 2x = -2 \sin(2x + h) \sin h$ (using $\cos C - \cos D = -2 \sin((C + D)/2) \sin((C - D)/2)$). **[2]**

$[f(x + h) - f(x)]/h = -2 \sin(2x + h) \times (\sin h / h)$. **[1]**

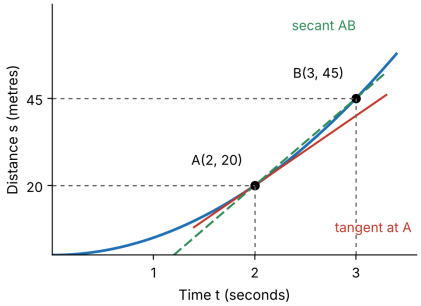
As $h \to 0$: $\sin h / h \to 1$ and $\sin(2x + h) \to \sin 2x$, so $f'(x) =$ **$-2 \sin 2x$**. **[1]**

At $x = \pi/4$: $f'(\pi/4) = -2 \sin(\pi/2) =$ **-2**. **[1]**

SECTION – E (1 × 4 = 4)

Case Study Based Question

Q23. **4 marks**



(i) s(3) = 45 m and s(2) = 20 m. Average velocity = (45 – 20)/(3 – 2) = **25 m/s**. **[1]**

(ii) Average velocity = [s(2 + h) – s(2)]/h = [5(2 + h)² – 20]/h = (20h + 5h²)/h = **20 + 5h** m/s. **[1]**

(iii)(a) $s'(t) = \lim_{h\to 0} [5(t + h)^2 – 5t^2]/h = \lim_{h\to 0} (10t + 5h) = 10t$. **[1]**

Velocity at t = 2 is $s'(2) =$ **20 m/s**. **[1]**

OR (b) The slope of the tangent at B (t = 3) = $s'(3) = 10 \times 3 =$ **30**. **[1]**

Slope = 50 ⇒ 10t = 50 ⇒ t = **5 s**. **[1]**

— End of Solutions —