CLASS XI – MATHEMATICS (041)

CHAPTER 2 : RELATIONS AND FUNCTIONS

PRACTICE PAPER – STEP-BY-STEP SOLUTIONS & MARKING SCHEME

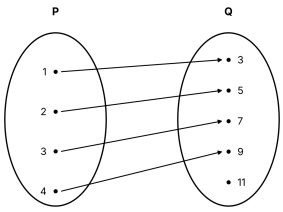
Section A – Answer Key (1 mark each)

Q1	Q2	Q3	Q4	Q5	Q6	Q7
B	D	C	A	C	B	D
Q8	**Q9**	**Q10**	**Q11**	**Q12**	**Q13**	**Q14**
A	C	A	B	D	A	C

SECTION – A (14 × 1 = 14)

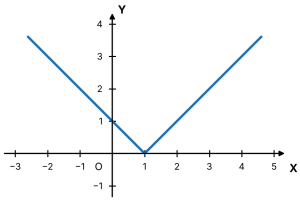
Multiple Choice and Assertion–Reason Questions

Q1. Answer: (B) — 1 mark



From Fig. 1 the ordered pairs are (1, 3), (2, 5), (3, 7), (4, 9); the element 11 of Q is not related to anything.
Each y is one more than twice x: 3 = 2(1)+1, 5 = 2(2)+1, 7 = 2(3)+1, 9 = 2(4)+1. So R = {(x, y) : y = 2x + 1, x ∈ P}.
Check the others: y = x + 2 gives (2, 4) ∉ R; y = 2x + 3 gives (1, 5) ∉ R; y = x^2 + 2 gives (2, 6) ∉ R.

Q2. Answer: (D) — 1 mark



The graph is a 'V' with its vertex at (1, 0), and it cuts the y-axis at (0, 1).
|x − 1| has vertex at x = 1 with value 0, and at x = 0 it equals 1. ✓ (|x| − 1 has vertex (0, −1); |x| + 1 has vertex (0, 1); |x + 1| has vertex (−1, 0).)

Q3. Answer: (C) — 1 mark

n(A × B) = 3 × 2 = 6. Every subset of A × B is a relation, so the number of relations = 2^6 = **64**.

Q4. Answer: (A) — 1 mark

Equal ordered pairs have equal corresponding elements: 2x − 1 = 5 ⇒ x = 3.
y + 3 = 2y − 1 ⇒ y = 4. So x + y = **7**.

Q5. Answer: (C) — 1 mark

n(A × B) = n(A) · n(B) ⇒ 12 = 3 · n(B) ⇒ n(B) = 4.
n(B × A × A) = 4 × 3 × 3 = **36**.

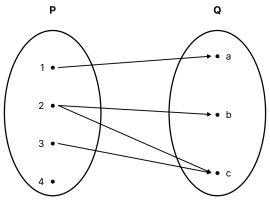
Q6. Answer: (B) — 1 mark

A function needs every element of A to have exactly one image.
(A) 1 has two images (a and c) – not a function. (C) 3 has no image – not a function. (D) the first elements a, b, c are not in A, so it is not even a relation from A to B.
(B) 1, 2, 3 each have exactly one image (a) – a function.

Q7. Answer: (D) — 1 mark

f is defined when the denominator is non-zero. x^2 − 4x − 5 = (x − 5)(x + 1) = 0 at x = 5 and x = −1.
Domain = R − {−1, 5}.

Q8. Answer: (A) **1 mark**



In Fig. 3 the pairs are (1, a), (2, b), (2, c), (3, c).
Element 2 is joined to both b and c (two images), and element 4 has no arrow (no image). Both conditions of a function are violated, so option (A).

Q9. Answer: (C) **1 mark**

$|x| \geq 0$ for all x, so $2 - |x| \leq 2$, and the value 2 is reached at $x = 0$.
Every $y \leq 2$ occurs (take $x = \pm(2 - y)$). Range = $(-\infty, 2]$.

Q10. Answer: (A) **1 mark**

$[-2.3] = -3$, $[3.9] = 3$, $[-0.5] = -1$.
$[-2.3] + [3.9] - [-0.5] = -3 + 3 - (-1) =$ **1**.

Q11. Answer: (B) **1 mark**

$f(-4) = -1$ (since $-4 < 0$); $f(0) = 0$; $f(2.5) = 1$ (since $2.5 > 0$).
$f(-4) + f(0) - f(2.5) = -1 + 0 - 1 =$ **−2**.

Q12. Answer: (D) **1 mark**

$(f - g)(-1) = f(-1) - g(-1) = (-1)^2 - (2(-1) + 3) = 1 - 1 = 0$.
$(fg)(1) = f(1) \cdot g(1) = 1 \times 5 = 5$.
Sum = 0 + 5 = **5**.

Q13. Answer: (A) **1 mark**

$A \times B = \{(1, 3), (2, 3)\}$ and $B \times A = \{(3, 1), (3, 2)\}$. These are not equal, so Assertion is true.
Two ordered pairs are equal only when corresponding elements are equal; (a, b) = (b, a) needs a = b. So Reason is true.
Since $1 \neq 3$, the pairs (1, 3) and (3, 1) are not equal, which is exactly what the Reason says; so R explains A → option (A).

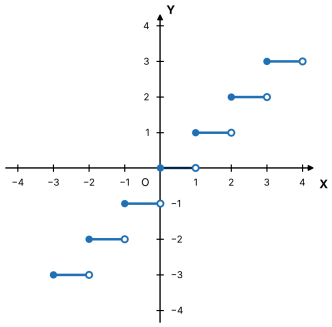
Q14. Answer: (C) **1 mark**

A function is a special relation in which every element of the domain has exactly one image, so every function is a relation. Assertion is true.
Not every relation is a function: $R = \{(1, a), (1, b)\}$ is a relation but 1 has two images. Reason is false → option (C).

SECTION – B (4 × 2 = 8)

Very Short Answer Questions

Q15. 2 marks



(i) [x] is the greatest integer not exceeding x. From the graph, for $-2 \le x < -1$ the value is -2, so $f(-1.5) = \mathbf{-2}$. For $2 \le x < 3$ the value is 2, so $f(2.9) = \mathbf{2}$. **[1]**

(ii) The graph takes only integer values, and every integer is attained. Range = $\mathbf{Z}$ (the set of all integers). **[1]**

Q16. 2 marks

$A \times B = \{(2, 3), (2, 4), (3, 3), (3, 4)\}$. **[½]**

$B \times A = \{(3, 2), (3, 3), (4, 2), (4, 3)\}$. **[½]**

The only pair common to both is (3, 3). So $(A \times B) \cap (B \times A) = \mathbf{\{(3, 3)\}}$. **[1]**

Q17. 2 marks

(A) $y = 2x$ with $x, y \in A = \{1, \ldots, 6\}$: $x = 1 \to 2$, $x = 2 \to 4$, $x = 3 \to 6$; for $x = 4, 5, 6$ the value $2x$ is not in A.

$R = \mathbf{\{(1, 2), (2, 4), (3, 6)\}}$. **[1]**

Domain = **{1, 2, 3}**; Range = **{2, 4, 6}**. **[1]**

OR (B) Primes less than 10: 2, 3, 5, 7. Cubes: 8, 27, 125, 343.

$R = \mathbf{\{(2, 8), (3, 27), (5, 125), (7, 343)\}}$. **[1]**

Domain = **{2, 3, 5, 7}**; Range = **{8, 27, 125, 343}**. **[1]**

Q18. 2 marks

(A) f is real only if $25 - x^2 \ge 0$, i.e. $x^2 \le 25$, i.e. $-5 \le x \le 5$. Domain = $\mathbf{[-5, 5]}$. **[1]**

For x in $[-5, 5]$, $0 \le 25 - x^2 \le 25$ (maximum at $x = 0$), so $0 \le f(x) \le 5$, and every value in between is attained. Range = **[0, 5]**. **[1]**

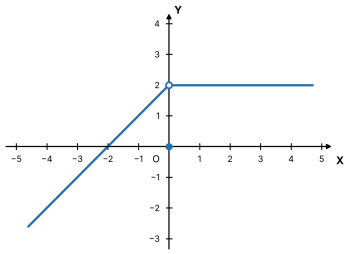
OR (B) The denominator $x^2 - x - 6 = (x - 3)(x + 2)$ is zero when $x = 3$ or $x = -2$. **[1]**

Domain = $\mathbf{R - \{-2, 3\}}$ (both values must be excluded, even though $(x + 2)$ appears in the numerator). **[1]**

SECTION – C (3 × 3 = 9)

Short Answer Questions

Q19. 3 marks



(i) For $x < 0$ the graph is the line through $(-2, 0)$ and $(0, 2)$ (open dot at $(0, 2)$), i.e. $y = x + 2$. At $x = 0$ the point is $(0, 0)$. For $x > 0$ the graph is the horizontal line $y = 2$. **[½]**

$f(x) =$ **$x + 2$, if $x < 0$; 0, if $x = 0$; 2, if $x > 0$**. **[½]**

(ii) $f(-3) = -3 + 2 = -1$; $f(0) = 0$; $f(5) = 2$. Sum $= -1 + 0 + 2 =$ **1**. **[1]**

(iii) For $x < 0$, $x + 2$ takes all values less than 2; at $x = 0$ the value is 0 (already included); for $x > 0$ the value is 2. Range = **$(-\infty, 2]$**. **[1]**

Q20. 3 marks

(A) $B \cup C = \{2, 3, 4\}$. $A \times (B \cup C) = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)\}$. **[1]**

$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3)\}$; $A \times C = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$. **[1]**

$(A \times B) \cup (A \times C) = \{(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4)\}$. Observation: $A \times (B \cup C) = (A \times B) \cup (A \times C)$. **[1]**

OR (B) A is the set of first elements and B the set of second elements: A = **{p, q}**, B = **{x, y}**. **[1½]**

$A \times A =$ **$\{(p, p), (p, q), (q, p), (q, q)\}$**. **[1½]**

Q21. 3 marks

(A) $(f + g)(x) = x^2 - 4 + 2x + 1 =$ **$x^2 + 2x - 3$**. **[¾]**

$(f - g)(x) = x^2 - 4 - (2x + 1) =$ **$x^2 - 2x - 5$**. **[¾]**

$(fg)(x) = (x^2 - 4)(2x + 1) =$ **$2x^3 + x^2 - 8x - 4$**. **[¾]**

$(f/g)(x) = (x^2 - 4)/(2x + 1)$, defined when $g(x) \neq 0$, i.e. $x \neq -1/2$. Domain = **$\mathbf{R} - \{-1/2\}$**. **[¾]**

OR (B) $f(x) = 3 - |x + 2|$ is defined for every real x, so domain = **R**. **[1]**

$|x + 2| \geq 0$, so $f(x) = 3 - |x + 2| \leq 3$, with the value 3 at $x = -2$. Any $y \leq 3$ is obtained when $|x + 2| = 3 - y \geq 0$. Range = **$(-\infty, 3]$**. **[1]**

$f(-5) = 3 - |-3| = 0$ and $f(1) = 3 - |3| = 0$. **[1]**

SECTION – D **(1 × 5 = 5)**

Long Answer Question

Q22. **5 marks**

(A) A = {1, 2, 3, 4}; B = {1, 4, 9, 16, 25}; $y = x^2$.

(i) R = **{(1, 1), (2, 4), (3, 9), (4, 16)}**. **[1]**

(ii) Domain = {1, 2, 3, 4}; Codomain = B = {1, 4, 9, 16, 25}; Range = {1, 4, 9, 16}. **[1]**

(iii) Yes. Every element of A (the domain) appears as the first element of exactly one ordered pair, so each has one and only one image in B. Hence R is a function. **[1]**

(iv) n(A × B) = 4 × 5 = **20**. **[1]**

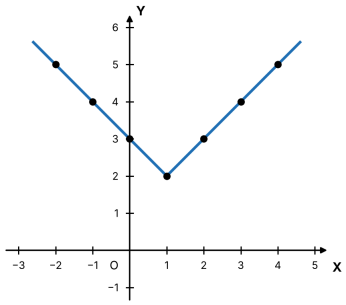
The number of relations = number of subsets of A × B = 2^{20} = **1,048,576**. **[1]**

OR (B) (i) If $x - 1 \geq 0$ ($x \geq 1$): $|x - 1| = x - 1$, so $f(x) = x + 1$. If $x < 1$: $|x - 1| = 1 - x$, so $f(x) = 3 - x$. f(x) = **$x + 1, x \geq 1$; $3 - x, x < 1$**. **[1]**

(ii) Table of values (completed): **[1]**

x	−2	−1	0	1	2	3	4
f(x) = |x − 1| + 2	5	4	3	2	3	4	5

(iii) Plot the points (−2, 5), (−1, 4), (0, 3), (1, 2), (2, 3), (3, 4), (4, 5) and join them: a 'V'-shaped graph with vertex (1, 2) (plotting of points ½, correct shape with vertex ½ + ½, labelled axes ½). **[2]**

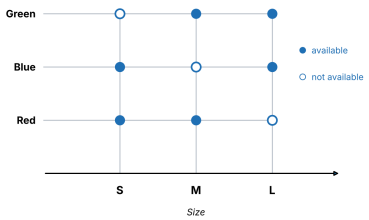


(iv) Domain = **R**; since $|x - 1| \geq 0$, $f(x) \geq 2$ with value 2 at x = 1, so Range = **[2, ∞)**. **[1]**

SECTION – E (1 × 4 = 4)

Case Study Based Question

Q23. **4 marks**



From Fig. 6: A = {S, M, L}, B = {Red, Blue, Green}. The filled (in-stock) points are: (S, Red), (S, Blue), (M, Red), (M, Green), (L, Blue), (L, Green).

(i) n(A × B) = 3 × 3 = **9**. **[1]**

(ii) R = **{(S, Red), (S, Blue), (M, Red), (M, Green), (L, Blue), (L, Green)}**. **[1]**

(iii)(a) No, R is not a function from A to B: the size S is related to two colours (Red and Blue); similarly M and L each have two images. So elements of the domain do not have a unique image. **[1]**

Domain = {S, M, L}; Range = {Red, Blue, Green}. **[1]**

OR (b) n(A × B) = 9, so the number of relations from A to B = 2^9 = **512**. **[1]**

n(A × A × B) = 3 × 3 × 3 = **27**. **[1]**

— End of Solutions —