CLASS XI – MATHEMATICS (041)

CHAPTER 3 : TRIGONOMETRIC FUNCTIONS

PRACTICE PAPER – STEP-BY-STEP SOLUTIONS & MARKING SCHEME

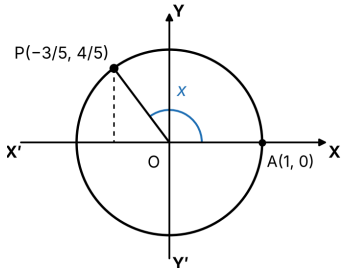
Section A – Answer Key (1 mark each)

Q1	Q2	Q3	Q4	Q5	Q6	Q7
B	C	A	D	A	C	D
Q8	**Q9**	**Q10**	**Q11**	**Q12**	**Q13**	**Q14**
B	A	B	C	D	D	B

SECTION – A (14 × 1 = 14)

Multiple Choice and Assertion–Reason Questions

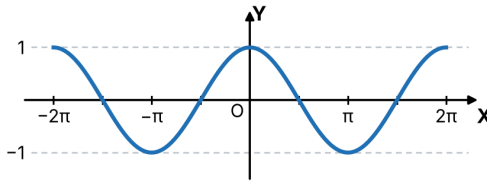
Q1. Answer: (B) — 1 mark



For a point P(a, b) on the unit circle with ∠AOP = x: cos x = a and sin x = b. Here cos x = −3/5 and sin x = 4/5.

tan x = sin x / cos x = (4/5) ÷ (−3/5) = **−4/3**.

Q2. Answer: (C) — 1 mark



The graph has its maximum value 1 at x = 0, cuts the x-axis at ±π/2, ±3π/2 and repeats after 2π. This is the graph of cos x.

sin(π/2 + x) = cos x (identity). ✓

Check the others: sin(π − x) = sin x passes through the origin; cos(π − x) = −cos x has value −1 at x = 0; sin(2π − x) = −sin x passes through the origin.

Q3. Answer: (A) — 1 mark

37°30′ = 37½° = 75/2 degrees.

Radian measure = (π/180) × (75/2) = 75π/360 = **5π/24**.

Q4. Answer: (D) — 1 mark

θ = l / r = 9 / 6 = **3/2** radian.

Q5. Answer: (A) — 1 mark

cos is even and has period 2π (360°): cos(−1590°) = cos(1590°).

1590° = 4 × 360° + 150°, so cos(1590°) = cos 150° = cos(180° − 30°) = −cos 30° = **−√3/2**.

Q6. Answer: (C) — 1 mark

tan x > 0 in quadrants I and III. cos x < 0 in quadrants II and III.

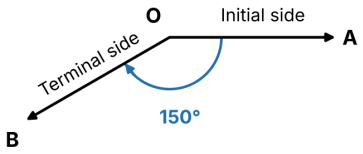
The common quadrant is the **third quadrant**.

Q7. Answer: (D) — 1 mark

sec(π/4) = √2 ⇒ $\sec^2(\pi/4) = 2$; cosec(π/6) = 2 ⇒ $\text{cosec}^2(\pi/6) = 4$; cot(π/4) = 1 ⇒ $\cot^2(\pi/4) = 1$.

Value = 2 + 4 − 1 = **5**.

Q8. Answer: (B) **1 mark**



The arrow in Fig. 3 shows a **clockwise** rotation, so the angle is negative: −150°.
−150° = −150 × π/180 = **−5π/6** radian.

Q9. Answer: (A) **1 mark**

cosec x = 1 / sin x, and sin x takes every value in [−1, 1] except 0 (where cosec x is not defined).
So |sin x| ≤ 1 gives |cosec x| ≥ 1. Range = **(−∞, −1] ∪ [1, ∞)**.

Q10. Answer: (B) **1 mark**

sin 40° cos 20° + cos 40° sin 20° = sin(40° + 20°) = sin 60° = **√3/2**.

Q11. Answer: (C) **1 mark**

tan 2x = 2 tan x / (1 − $\tan^2$ x) = (2/3) / (1 − 1/9) = (2/3) / (8/9) = (2/3)(9/8) = **3/4**.

Q12. Answer: (D) **1 mark**

cos x − cos y = −2 sin((x + y)/2) sin((x − y)/2). With x = 75°, y = 15°:
cos 75° − cos 15° = −2 sin 45° sin 30° = −2 × (1/√2) × (1/2) = **−1/√2**.

Q13. Answer: (D) **1 mark**

tan x = sin x / cos x is not defined when cos x = 0, i.e. at x = (2n + 1)π/2, n ∈ Z. So the domain is R − {(2n + 1)π/2 : n ∈ Z}, not R. Assertion is false.
Reason is true: cos x = 0 exactly at odd multiples of π/2, where tan x is undefined. → option (D).

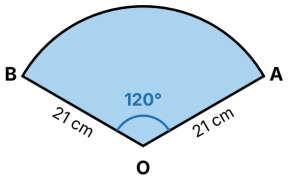
Q14. Answer: (B) **1 mark**

sin 120° = sin(180° − 60°) = sin 60° = √3/2. Assertion is true.
$\sin^2$ x + $\cos^2$ x = 1 holds for all real x (from a^2 + b^2 = 1 on the unit circle). Reason is true.
But this identity does not explain why sin(π − x) = sin x (that comes from the sine-difference formula).
So R is not the correct explanation → option (B).

SECTION – B (4 × 2 = 8)

Very Short Answer Questions

Q15. 2 marks



(i) Radian measure = (π/180) × degree measure. ∠AOB = 120° = 120π/180 = **2π/3 radian**. **[1]**
(ii) Arc length l = rθ = 21 × 2π/3 = 14π cm. **[½]**
With π = 22/7: l = 14 × 22/7 = **44 cm**. **[½]**

Q16. 2 marks

$\sin^2 x = 1 - \cos^2 x = 1 - 25/169 = 144/169$, so $\sin x = \pm 12/13$. **[½]**
In the third quadrant sin x is negative. Hence sin x = **−12/13**. **[½]**
tan x = sin x / cos x = (−12/13) ÷ (−5/13) = **12/5** (positive, as expected in quadrant III). **[1]**

Q17. 2 marks

(A) cos 15° = cos(45° − 30°) = cos 45° cos 30° + sin 45° sin 30°. **[1]**
= (1/√2)(√3/2) + (1/√2)(1/2) = **(√3 + 1) / (2√2)**. **[1]**
OR (B) sin is odd: sin(−1125°) = −sin(1125°). **[1]**
1125° = 3 × 360° + 45°, so sin(1125°) = sin 45° = 1/√2. Hence sin(−1125°) = **−1/√2**. **[1]**

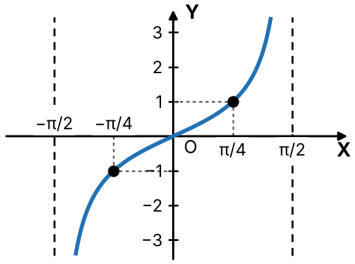
Q18. 2 marks

(A) LHS = $(1 + \cos 2x)/\sin 2x = 2\cos^2 x / (2 \sin x \cos x)$ [using $1 + \cos 2x = 2\cos^2 x$ and $\sin 2x = 2 \sin x \cos x$]. **[1]**
= cos x / sin x = cot x = RHS. (Hence proved.) **[1]**
OR (B) LHS = $(\sin x + \cos x)^2 = \sin^2 x + \cos^2 x + 2 \sin x \cos x$. **[1]**
= 1 + sin 2x = RHS. (Hence proved.) **[1]**

SECTION – C (3 × 3 = 9)

Short Answer Questions

Q19. 3 marks



(i) The graph passes through the points where the dotted lines meet the curve: (π/4, 1) and (−π/4, −1). So tan(π/4) = **1** and tan(−π/4) = **−1**. **[1]**

(ii) Reading the graph from left to right, the curve keeps rising from very large negative values near x = −π/2 to very large positive values near x = π/2. So tan x is **increasing** on (−π/2, π/2), and its range is **R**. **[1]**

(iii) tan 75° = tan(45° + 30°) = (tan 45° + tan 30°) / (1 − tan 45° tan 30°) = (1 + 1/√3) / (1 − 1/√3) = (√3 + 1)/(√3 − 1). **[½]**

Rationalising: $(\sqrt{3}+1)^2 / (3-1) = (4+2\sqrt{3})/2 =$ **2 + √3**. **[½]**

Q20. 3 marks

(A) Numerator: cos 2x + cos 6x = 2 cos((6x + 2x)/2) cos((6x − 2x)/2) = 2 cos 4x cos 2x. **[1]**

Denominator: sin 6x − sin 2x = 2 cos((6x + 2x)/2) sin((6x − 2x)/2) = 2 cos 4x sin 2x. **[1]**

LHS = (2 cos 4x cos 2x) / (2 cos 4x sin 2x) = cos 2x / sin 2x = cot 2x = RHS. **[1]**

OR (B) Numerator: 1 + sin 2x − cos 2x = (1 − cos 2x) + sin 2x = $2\sin^2 x + 2 \sin x \cos x = 2 \sin x (\sin x + \cos x)$. **[1]**

Denominator: 1 + sin 2x + cos 2x = (1 + cos 2x) + sin 2x = $2\cos^2 x + 2 \sin x \cos x = 2 \cos x (\cos x + \sin x)$. **[1]**

LHS = [2 sin x (sin x + cos x)] / [2 cos x (sin x + cos x)] = sin x / cos x = tan x = RHS. **[1]**

Q21. 3 marks

(A) x in quadrant II: $\cos^2 x = 1 - 64/289 = 225/289$, so cos x = −15/17 (cos is negative in II). **[1]**

y in quadrant IV: $\sin^2 y = 1 - 9/25 = 16/25$, so sin y = −4/5 (sin is negative in IV). **[1]**

cos(x − y) = cos x cos y + sin x sin y = (−15/17)(3/5) + (8/17)(−4/5) = −45/85 − 32/85 = **−77/85**. **[1]**

OR (B) x in quadrant II: $\sec^2 x = 1 + \tan^2 x = 1 + 25/144 = 169/144$, so cos x = −12/13 (negative in II). **[½]**

sin x = tan x · cos x = (−5/12)(−12/13) = 5/13. **[½]**

sin 2x = 2 sin x cos x = 2 × (5/13) × (−12/13) = **−120/169**. **[1]**

cos 2x = $\cos^2 x - \sin^2 x$ = 144/169 − 25/169 = **119/169**. **[1]**

SECTION – D (1 × 5 = 5)

Long Answer Question

Q22. **5 marks**

(A) From $\cos 3x = 4\cos^3 x - 3\cos x$ we get $\cos^3 x = (\cos 3x + 3\cos x)/4$. **[1]**

From $\sin 3x = 3\sin x - 4\sin^3 x$ we get $\sin^3 x = (3\sin x - \sin 3x)/4$. **[1]**

LHS $= [(\cos 3x + 3\cos x)/4]\sin 3x + [(3\sin x - \sin 3x)/4]\cos 3x$. **[1]**

$= (1/4)[\sin 3x \cos 3x + 3\sin 3x \cos x + 3\sin x \cos 3x - \sin 3x \cos 3x]$ **[1]**

$= (3/4)[\sin 3x \cos x + \cos 3x \sin x] = (3/4)\sin(3x + x) =$ **$(3/4)\sin 4x$** = RHS. (Hence proved.) **[1]**

OR (B) Since $\pi < x < 3\pi/2$, we get $\pi/2 < x/2 < 3\pi/4$: $x/2$ lies in the second quadrant, so $\sin(x/2) > 0$ and $\cos(x/2) < 0$. **[1]**

Use $\cos x = 1 - 2\sin^2(x/2) = 2\cos^2(x/2) - 1$. **[1]**

$2\sin^2(x/2) = 1 - \cos x = 1 + 7/25 = 32/25 \Rightarrow \sin^2(x/2) = 16/25 \Rightarrow \sin(x/2) =$ **$4/5$**. **[1]**

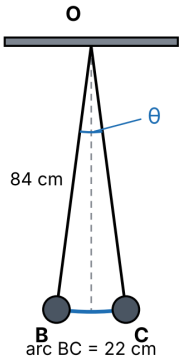
$2\cos^2(x/2) = 1 + \cos x = 1 - 7/25 = 18/25 \Rightarrow \cos^2(x/2) = 9/25 \Rightarrow \cos(x/2) =$ **$-3/5$**. **[1]**

$\tan(x/2) = \sin(x/2) / \cos(x/2) = (4/5) \div (-3/5) =$ **$-4/3$**. **[1]**

SECTION – E (1 × 4 = 4)

Case Study Based Question

Q23. **4 marks**



Radius r = 84 cm, arc length l = 22 cm, π = 22/7.

(i) θ = l / r = 22/84 = **11/42 radian**. **[1]**

(ii) θ in degrees = (180/π) × (11/42) = 180 × (7/22) × (11/42) = **15°**. **[1]**

(iii)(a) 45° = 45 × π/180 = π/4 radian. **[1]**

Arc length l = rθ = 84 × π/4 = 21π = 21 × 22/7 = **66 cm**. **[1]**

OR (b) θ = 15° = 45° − 30°, so sin 15° = sin 45° cos 30° − cos 45° sin 30°. **[1]**

= (1/√2)(√3/2) − (1/√2)(1/2) = **(√3 − 1) / (2√2)**. **[1]**

— End of Solutions —