CLASS XI – MATHEMATICS (041)

CHAPTER 4 : COMPLEX NUMBERS AND QUADRATIC EQUATIONS

PRACTICE PAPER – STEP-BY-STEP SOLUTIONS & MARKING SCHEME

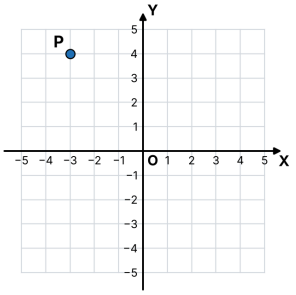
Section A – Answer Key (1 mark each)

Q1	Q2	Q3	Q4	Q5	Q6	Q7
B	C	D	A	B	C	D
Q8	**Q9**	**Q10**	**Q11**	**Q12**	**Q13**	**Q14**
A	B	C	A	D	C	D

SECTION – A (14 × 1 = 14)

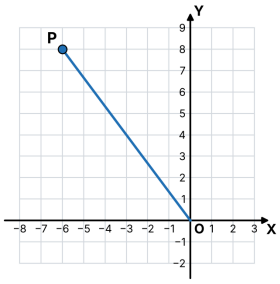
Multiple Choice and Assertion–Reason Questions

Q1. Answer: (B) — 1 mark



A point (x, y) in the Argand plane represents x + iy. From Fig. 1, P is (−3, 4), so z = −3 + 4i.
The conjugate of a + ib is a − ib. Hence $\bar{z}$ = **−3 − 4i**.

Q2. Answer: (C) — 1 mark



From Fig. 2, P is (−6, 8), so z = −6 + 8i.
$|z| = \sqrt{((-6)^2 + 8^2)} = \sqrt{(36 + 64)} = \sqrt{100}$ = **10** (the distance OP).

Q3. Answer: (D) — 1 mark

For any integer k: $i^{4k} = 1$, $i^{4k+1} = i$, $i^{4k+2} = -1$, $i^{4k+3} = -i$.
43 = 4(10) + 3, so $i^{43} = -i$. Also −21 = 4(−6) + 3, so $i^{-21} = -i$.
Sum = −i + (−i) = **−2i**.

Q4. Answer: (A) — 1 mark

$z = (2 - i)(3 + 4i) = 6 + 8i - 3i - 4i^2 = 6 + 4 + 5i = 10 + 5i$.
Re z = 10 and Im z = 5, so Re z + Im z = **15**.

Q5. Answer: (B) — 1 mark

Equating real and imaginary parts: x + 2y = 4 …(1) and 3x − y = 5 …(2).
From (2): y = 3x − 5. Put in (1): x + 6x − 10 = 4 ⇒ 7x = 14 ⇒ x = 2, and y = 1.
$x^2 + y^2 = 4 + 1$ = **5**.

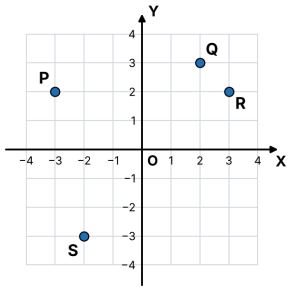
Q6. Answer: (C) — 1 mark

For z = a + ib, $z^{-1} = \bar{z} / |z|^2$. Here $\bar{z} = 5 + 12i$ and $|z|^2 = 25 + 144 = 169$.
Multiplicative inverse = **(5 + 12i)/169**.

Q7. Answer: (D) — 1 mark

$|z_1 z_2| = |z_1||z_2| = |2 + i| \cdot |1 - 2i| = \sqrt{5} \cdot \sqrt{5}$ = **5**.
(Check: $(2 + i)(1 - 2i) = 2 - 4i + i - 2i^2 = 4 - 3i$, and $|4 - 3i| = \sqrt{(16 + 9)} = 5$.)

Q8. Answer: (A) **1 mark**



$i(2 + 3i) = 2i + 3i^2 = -3 + 2i$, which corresponds to the point $(-3, 2)$.
From Fig. 3, the point $(-3, 2)$ is **P**. (R(3, 2) would represent $3 + 2i$, Q(2, 3) represents $2 + 3i$ itself, and S$(-2, -3)$ represents $-2 - 3i$.)

Q9. Answer: (B) **1 mark**

$\sqrt{-4} = 2i$ and $\sqrt{-9} = 3i$.
Product $= 6i^2 = \mathbf{-6}$. (Note: $\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$ is *not* valid when both a and b are negative; otherwise we would wrongly get $\sqrt{36} = 6$.)

Q10. Answer: (C) **1 mark**

$(1 + i)^2 = 1 + 2i + i^2 = 2i$.
$2i/(2 - i) = 2i(2 + i)/(4 + 1) = (4i + 2i^2)/5 = (-2 + 4i)/5$.
Conjugate = $\mathbf{(-2 - 4i)/5}$.

Q11. Answer: (A) **1 mark**

$(1 + i)^2 = 2i$, so $(1 + i)^6 = (2i)^3 = 8i^3 = \mathbf{-8i}$.

Q12. Answer: (D) **1 mark**

$(3 + 2i)/(1 - i) = (3 + 2i)(1 + i)/(1 + 1) = (3 + 3i + 2i + 2i^2)/2 = (1 + 5i)/2$.
Imaginary part = $\mathbf{5/2}$.

Q13. Answer: (C) **1 mark**

$(3i)^2 = 9i^2 = -9$ and $(-3i)^2 = -9$, so the square roots of -9 are $3i$ and $-3i$. Assertion is true.
$\sqrt{a} \times \sqrt{b} = \sqrt{(ab)}$ holds when at least one of a, b is non-negative, but fails when both are negative (e.g. $\sqrt{-1} \times \sqrt{-1} = i^2 = -1$, whereas $\sqrt{1} = 1$). Reason is false → option (C).

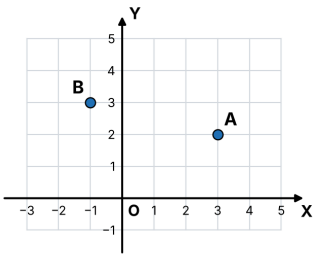
Q14. Answer: (D) **1 mark**

The multiplicative inverse of $3 - 4i$ is $\bar{z}/|z|^2 = (3 + 4i)/25$, not $3 + 4i$. (Indeed $(3 - 4i)(3 + 4i) = 25 \neq 1$.) Assertion is false.
Reason is the standard formula for the multiplicative inverse, so it is true → option (D).

SECTION – B (4 × 2 = 8)

Very Short Answer Questions

Q15. 2 marks



(i) From Fig. 4: A is (3, 2) and B is (−1, 3). So z_1 = **3 + 2i** and z_2 = **−1 + 3i**. **[1]**
(ii) $z_1z_2 = (3 + 2i)(-1 + 3i) = -3 + 9i - 2i + 6i^2 = -3 - 6 + 7i$ = **−9 + 7i**. **[1]**

Q16. 2 marks

$i^{37} = i^{4(9)+1} = i$. **[½]**
$i^{67} = i^{4(16)+3} = -i$, so $1/i^{67} = 1/(-i) = -1/i = -(-i) = i$. (Since $1/i = i/i^2 = -i$.) **[1]**
Sum = i + i = **2i**. **[½]**

Q17. 2 marks

(A) Multiply numerator and denominator by the conjugate of the denominator: (2 + 3i)(4 + i) / [(4 − i)(4 + i)]. **[1]**
$= (8 + 2i + 12i + 3i^2)/(16 + 1) = (5 + 14i)/17$ = **5/17 + (14/17)i**. **[1]**
OR (B) z = 2 − 5i, $\bar{z}$ = 2 + 5i, $|z|^2 = 4 + 25 = 29$. **[1]**
$z^{-1} = \bar{z}/|z|^2$ = **(2 + 5i)/29** = 2/29 + (5/29)i. **[1]**

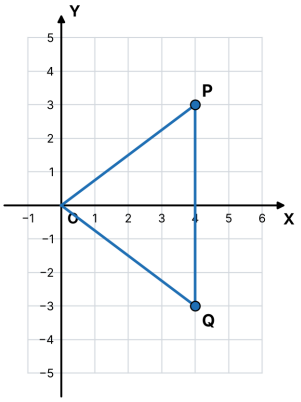
Q18. 2 marks

(A) $(1 + 2i)/(1 - 3i) = (1 + 2i)(1 + 3i)/(1 + 9) = (1 + 3i + 2i + 6i^2)/10 = (-5 + 5i)/10 = -1/2 + (1/2)i$. **[1]**
Conjugate = **−1/2 − (1/2)i**; Modulus = √(1/4 + 1/4) = **1/√2**. **[1]**
OR (B) x + ix + 2y − iy = 5 + 2i ⇒ (x + 2y) + i(x − y) = 5 + 2i. **[1]**
x + 2y = 5 and x − y = 2. Subtracting: 3y = 3 ⇒ **y = 1**, and **x = 3**. **[1]**

SECTION – C (3 × 3 = 9)

Short Answer Questions

Q19. 3 marks



(i) From Fig. 5, P is (4, 3) and Q is (4, −3). So $z = \mathbf{4 + 3i}$ and $\bar{z} = \mathbf{4 - 3i}$. **[1]**

(ii) $|z| = \sqrt{(16 + 9)} = \mathbf{5}$ and $|\bar{z}| = \sqrt{(16 + 9)} = \mathbf{5}$ (equal, as P and Q are equidistant from O). **[1]**

(iii) $z\,\bar{z} = (4 + 3i)(4 - 3i) = 16 - 9i^2 = 16 + 9 = \mathbf{25} = 5^2 = |z|^2$. Hence $z\,\bar{z} = |z|^2$. **[1]**

Q20. 3 marks

(A) $(2 + i)/(3 - i) = (2 + i)(3 + i)/10 = (6 + 2i + 3i + i^2)/10 = (5 + 5i)/10$. **[1]**

$(2 - i)/(3 + i) = (2 - i)(3 - i)/10 = (6 - 2i - 3i + i^2)/10 = (5 - 5i)/10$. **[1]**

Sum $= (5 + 5i + 5 - 5i)/10 = 10/10 = \mathbf{1 + 0i}$. **[1]**

OR (B) $(1 + i)^2 = 2i$, so $(1 + i)^3 = 2i(1 + i) = 2i + 2i^2 = -2 + 2i$. **[1]**

$(-2 + 2i)/(2 - i) = (-2 + 2i)(2 + i)/(4 + 1) = (-4 - 2i + 4i + 2i^2)/5$. **[1]**

$= (-6 + 2i)/5 = \mathbf{-6/5 + (2/5)i}$. **[1]**

Q21. 3 marks

(A) $z_1z_2 = (1 + 2i)(3 - i) = 3 - i + 6i - 2i^2 = 5 + 5i$, so $|z_1z_2| = \sqrt{50} = 5\sqrt{2}$. **[1]**

$|z_1| = \sqrt{5}$, $|z_2| = \sqrt{10}$, so $|z_1||z_2| = \sqrt{50} = 5\sqrt{2}$. Hence $|z_1z_2| = |z_1||z_2|$. ✓ **[1]**

$z_1/z_2 = (1 + 2i)(3 + i)/10 = (1 + 7i)/10$, so $|z_1/z_2| = \sqrt{50}/10 = 1/\sqrt{2}$; and $|z_1|/|z_2| = \sqrt{5}/\sqrt{10} = 1/\sqrt{2}$. ✓ **[1]**

OR (B) Numerator: $(3 + i)(1 - 2i) = 3 - 6i + i - 2i^2 = 5 - 5i$. **[1]**

$z = (5 - 5i)/(2 + i) = (5 - 5i)(2 - i)/5 = (1 - i)(2 - i) = 2 - i - 2i + i^2 = 1 - 3i$. **[1]**

$\bar{z} = \mathbf{1 + 3i}$; $|z| = \sqrt{(1 + 9)} = \mathbf{\sqrt{10}}$. **[1]**

SECTION – D **(1 × 5 = 5)**

Long Answer Question

Q22. **5 marks**

(A) (i) $z = (3 + 4i)(2 + i) / [(2 - i)(2 + i)] = (6 + 3i + 8i + 4i^2)/(4 + 1)$ **[1]**

$= (2 + 11i)/5 =$ **$2/5 + (11/5)i$**. **[1]**

(ii) $\bar{z} =$ **$(2 - 11i)/5$**; $|z|^2 = (4 + 121)/25 = 5$, so $|z| =$ **$\sqrt{5}$**. (Check: $|3 + 4i|/|2 - i| = 5/\sqrt{5} = \sqrt{5}$.) **[1]**

(iii) $z^{-1} = \bar{z}/|z|^2 = (2 - 11i)/(5 \times 5) =$ **$(2 - 11i)/25$**. **[1]**

(iv) $z\,\bar{z} = (2 + 11i)(2 - 11i)/25 = (4 + 121)/25 = 5$, and $|z|^2 = 5$. Hence $z\,\bar{z} = |z|^2$. ✓ **[1]**

OR (B) (i) $(x + iy)(1 - 2i) = x - 2xi + iy - 2i^2y = (x + 2y) + i(y - 2x)$. **[1]**

Equating with $5 + 5i$: $x + 2y = 5$ ...(1) and $-2x + y = 5$ ...(2). **[1]**

From (1): $x = 5 - 2y$. Put in (2): $-10 + 4y + y = 5 \Rightarrow 5y = 15 \Rightarrow y = 3$, $x = -1$. So $x + iy =$ **$-1 + 3i$**. **[1]**

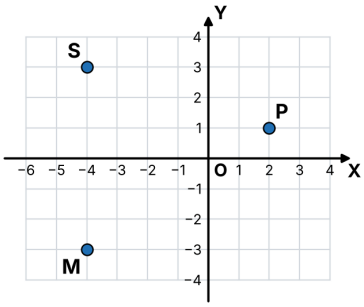
(ii) Conjugate of $-1 + 3i$ is **$-1 - 3i$**. **[1]**

Modulus $= \sqrt{(1 + 9)} =$ **$\sqrt{10}$**. **[1]**

SECTION – E (1 × 4 = 4)

Case Study Based Question

Q23. **4 marks**



From Fig. 6: S is (−4, 3), M is (−4, −3) and P is (2, 1).

(i) z_S = **−4 + 3i**; z_M = **−4 − 3i** (the conjugate of z_S). **[1]**

(ii) Distance of S from O = $|z_S|$ = √(16 + 9) = **5 km**. **[1]**

(iii)(a) $z_C = 1/z_S = \bar{z}_S/|z_S|^2$ = (−4 − 3i)/25. **[1]**

So the café is at **−4/25 − (3/25)i**. **[1]**

OR (b) z_P = 2 + i. z_S/z_P = (−4 + 3i)(2 − i)/[(2 + i)(2 − i)] = (−8 + 4i + 6i − $3i^2$)/5. **[1]**

= (−8 + 3 + 10i)/5 = (−5 + 10i)/5 = **−1 + 2i**. **[1]**

— End of Solutions —