CLASS XI – MATHEMATICS (041)

CHAPTER 6 : PERMUTATIONS AND COMBINATIONS

PRACTICE PAPER – STEP-BY-STEP SOLUTIONS & MARKING SCHEME

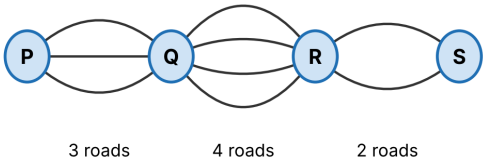
Section A – Answer Key (1 mark each)

Q1	Q2	Q3	Q4	Q5	Q6	Q7
B	D	A	C	A	C	D
Q8	**Q9**	**Q10**	**Q11**	**Q12**	**Q13**	**Q14**
B	C	A	D	B	B	A

SECTION – A (14 × 1 = 14)

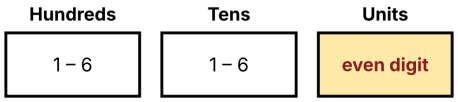
Multiple Choice and Assertion–Reason Questions

Q1. Answer: (B) — 1 mark



From the figure: P to Q – 3 roads, Q to R – 4 roads, R to S – 2 roads.
By the multiplication principle, number of ways = 3 × 4 × 2 = **24**.

Q2. Answer: (D) — 1 mark



Digits 1, 2, 3, 4, 5, 6 may be repeated

Fill the units place first: it must be even, so it can be 2, 4 or 6 → 3 ways.
Tens place: 6 ways (repetition allowed); hundreds place: 6 ways.
Number of such numbers = 6 × 6 × 3 = **108**.

Q3. Answer: (A) — 1 mark

$9!/(6! \cdot 3!) = (9 \times 8 \times 7 \times 6!)/(6! \times 6) = (9 \times 8 \times 7)/6 =$ **84**.

Q4. Answer: (C) — 1 mark

${}^{n}P_{2} = n(n-1) = 56 \Rightarrow n^2 - n - 56 = 0 \Rightarrow (n-8)(n+7) = 0$.
n cannot be negative, so n = **8**.

Q5. Answer: (A) — 1 mark

BANANA has 6 letters: A appears 3 times, N appears 2 times, B once.
Number of arrangements = $6!/(3! \cdot 2!) = 720/12 =$ **60**.

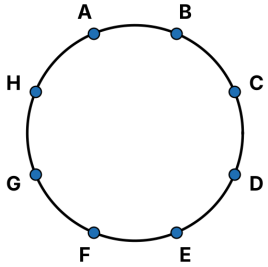
Q6. Answer: (C) — 1 mark

${}^{n}C_{2} = n(n-1)/2 = 45 \Rightarrow n(n-1) = 90 \Rightarrow n^2 - n - 90 = 0 \Rightarrow (n-10)(n+9) = 0$.
So n = **10** (n = –9 is rejected).

Q7. Answer: (D) — 1 mark

${}^{10}C_{r} = {}^{10}C_{r+2}$ means either r = r + 2 (impossible) or r + (r + 2) = 10.
$2r + 2 = 10 \Rightarrow r =$ **4**.

Q8. Answer: (B) **1 mark**



Any 3 of the 8 points (no three of which are collinear, being on a circle) form a triangle.
Number of triangles = ${}^{8}C_{3}$ = (8 × 7 × 6)/(3 × 2 × 1) = **56**.

Q9. Answer: (C) **1 mark**

A number is divisible by 5 if its units digit is 0 or 5.
Units digit 0: hundreds and tens are filled from 1, 2, 3, 4, 5 → ${}^{5}P_{2}$ = 5 × 4 = 20.
Units digit 5: hundreds digit can be 1, 2, 3 or 4 (not 0, not 5) → 4 ways; tens digit from the remaining 4 digits (0 now allowed) → 4 ways; 4 × 4 = 16.
Total = 20 + 16 = **36**.

Q10. Answer: (A) **1 mark**

Total committees of 3 from 7 persons = ${}^{7}C_{3}$ = 35.
Committees with no woman (all 3 men) = ${}^{4}C_{3}$ = 4.
At least one woman = 35 – 4 = **31**.

Q11. Answer: (D) **1 mark**

COMMITTEE has 9 letters: M appears 2 times, T appears 2 times, E appears 2 times; C, O, I once each.
Arrangements = 9!/(2! · 2! · 2!) = 362880/8 = **45360**.

Q12. Answer: (B) **1 mark**

Each of the 5 places can be filled in 5 ways (repetition allowed): 5^{5} = **3125**.

Q13. Answer: (B) **1 mark**

LEVEL has 5 letters: L appears 2 times, E appears 2 times, V once. Arrangements = 5!/(2! · 2!) = 120/4 = 30. Assertion is true.
Reason is also a true statement (${}^{n}P_{n}$ = n!). But it applies only when all n objects are different; LEVEL has repeated letters, so R does not explain A → option (B).

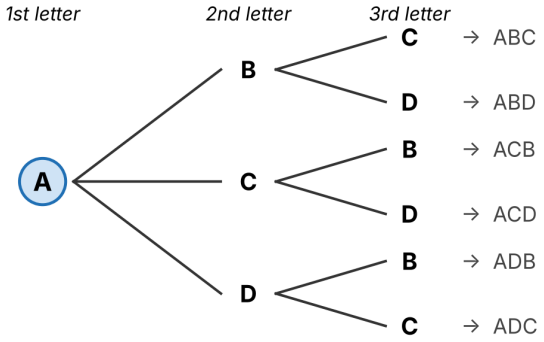
Q14. Answer: (A) **1 mark**

${}^{12}C_{9}$ = ${}^{12}C_{3}$ (taking n = 12, r = 9, n – r = 3). Both are equal to 220, so the Assertion is true.
Reason: ${}^{n}C_{r}$ = n!/[r!(n – r)!] = ${}^{n}C_{n-r}$, a true property, and it is exactly why A holds → option (A).

SECTION – B (4 × 2 = 8)

Very Short Answer Questions

Q15. 2 marks



(i) The tree lists the words ABC, ABD, ACB, ACD, ADB, ADC. Number of words beginning with A = **6**. **[1]**
(ii) The same count holds for each of the 4 possible first letters, so total = 4 × 6 = **24**. (Check: ${}^{4}P_{3}$ = 4 × 3 × 2 = 24.) **[1]**

Q16. 2 marks

$(n + 1)!/(n – 1)! = (n + 1) \cdot n \cdot (n – 1)!/(n – 1)! = n(n + 1)$. **[1]**
$n(n + 1) = 20 \Rightarrow n^2 + n – 20 = 0 \Rightarrow (n + 5)(n – 4) = 0 \Rightarrow n = –5$ or 4. Since n is a positive integer, **n = 4**. **[1]**

Q17. 2 marks

(A) PRINCE has 6 distinct letters, with vowels I and E. First place (vowel): 2 ways. **[1]**
Remaining 3 places from the other 5 letters: ${}^{5}P_{3} = 5 \times 4 \times 3 = 60$. Number of words = 2 × 60 = **120**. **[1]**
OR (B) ${}^{n}C_{10} = {}^{n}C_{14} \Rightarrow n = 10 + 14 = 24$ (since $10 \neq 14$). **[1]**
${}^{n}C_{22} = {}^{24}C_{22} = {}^{24}C_{2} = (24 \times 23)/2 = \mathbf{276}$. **[1]**

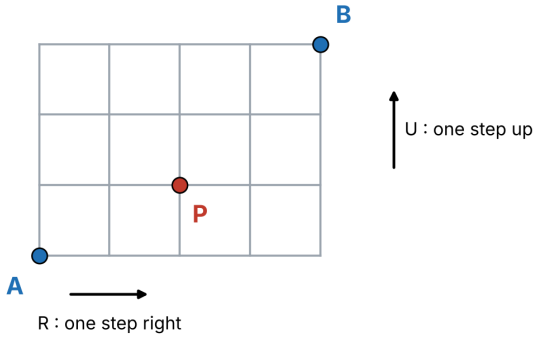
Q18. 2 marks

(A) 3 red balls out of 5: ${}^{5}C_{3} = 10$ ways; 2 blue balls out of 4: ${}^{4}C_{2} = 6$ ways. **[1]**
By the multiplication principle, number of ways = 10 × 6 = **60**. **[1]**
OR (B) A handshake is a selection of 2 persons out of 10 (order does not matter). **[1]**
Number of handshakes = ${}^{10}C_{2} = (10 \times 9)/2 = \mathbf{45}$. **[1]**

SECTION – C (3 × 3 = 9)

Short Answer Questions

Q19. 3 marks



(i) From the grid, A to B needs 4 steps to the right and 3 steps up. Every shortest route has **4 R-steps and 3 U-steps** (7 steps in all). **[1]**

(ii) A route is an arrangement of the 7 symbols R R R R U U U. Number of routes = 7!/(4! · 3!) = (7 × 6 × 5)/6 = **35**. **[1]**

(iii) A to P: 2 R and 1 U → 3!/(2! · 1!) = 3 routes. P to B: 2 R and 2 U → 4!/(2! · 2!) = 6 routes. Routes through P = 3 × 6 = **18**. **[1]**

Q20. 3 marks

(A) ENGINEERING has 11 letters: E – 3, N – 3, G – 2, I – 2, R – 1. **[½]**

Total arrangements = 11!/(3! · 3! · 2! · 2!) = 39916800/144 = **277200**. **[1½]**

Fix R at the first place and one G at the last place. The remaining 9 letters are E, E, E, N, N, N, G, I, I. **[½]**

Number of arrangements = 9!/(3! · 3! · 2!) = 362880/72 = **5040**. **[½]**

OR (B) The student answers 6 questions with at least 2 from each part. The possible splits (Part I, Part II) are (2, 4), (3, 3), (4, 2). **[1]**

(2, 4): ${}^5C_2 \times {}^5C_4 = 10 \times 5 = 50$; (3, 3): ${}^5C_3 \times {}^5C_3 = 10 \times 10 = 100$; (4, 2): ${}^5C_4 \times {}^5C_2 = 5 \times 10 = 50$. **[1½]**

Total = 50 + 100 + 50 = **200** ways. **[½]**

Q21. 3 marks

(A) There are 7 places in all; the even places are 2nd, 4th and 6th (3 places) and the odd places are 1st, 3rd, 5th and 7th (4 places). **[1]**

The 3 women can be seated in the 3 even places in 3! = 6 ways; the 4 men in the 4 odd places in 4! = 24 ways. **[1]**

Total = 6 × 24 = **144** ways. **[1]**

OR (B) In alphabetical order the letters are A, M, R, S, T. Words that begin with A, M or R (all come before S): 3 × 4! = 72. **[1]**

Next come the words starting with SA: the other 3 letters M, R, T can be arranged in 3! = 6 ways. So far 72 + 6 = 78 words. **[1]**

Words starting with SM: the first is SMART (letters A, R, T in alphabetical order). So SMART is the 78 + 1 = **79th word**. **[1]**

SECTION – D **(1 × 5 = 5)**

Long Answer Question

Q22. **5 marks**

(A) TRIANGLE has 8 different letters: 3 vowels (I, A, E) and 5 consonants (T, R, N, G, L). Total arrangements = 8! = 40320.

(i) Treat the 3 vowels as one object. This object with the 5 consonants gives 6 objects, which can be arranged in 6! = 720 ways. **[1]**

The vowels can be arranged among themselves in 3! = 6 ways. Number of arrangements = 720 × 6 = **4320**. **[1]**

(ii) First arrange the 5 consonants in 5! = 120 ways. This creates 6 gaps (including both ends): × C × C × C × C × C ×. **[1]**

The 3 vowels go into 3 different gaps: ${}^{6}P_{3}$ = 6 × 5 × 4 = 120 ways. Number of arrangements = 120 × 120 = **14400**. **[1]**

(iii) Fix T at the first place and E at the last place. The remaining 6 letters can be arranged in 6! = **720** ways. **[1]**

OR (B) 7 men and 6 women; committee of 6.

(i) Exactly 3 women, 3 men: ${}^{6}C_{3} \times {}^{7}C_{3}$ = 20 × 35 = **700**. **[1]**

(ii) At least 4 women: (4W, 2M) + (5W, 1M) + (6W, 0M) = ${}^{6}C_{4} \times {}^{7}C_{2} + {}^{6}C_{5} \times {}^{7}C_{1} + {}^{6}C_{6} \times {}^{7}C_{0}$. **[1]**

= 15 × 21 + 6 × 7 + 1 × 1 = 315 + 42 + 1 = **358**. **[1]**

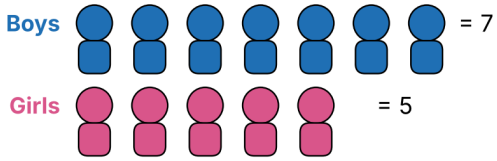
(iii) At most 2 women: (0W, 6M) + (1W, 5M) + (2W, 4M) = ${}^{6}C_{0} \times {}^{7}C_{6} + {}^{6}C_{1} \times {}^{7}C_{5} + {}^{6}C_{2} \times {}^{7}C_{4}$. **[1]**

= 1 × 7 + 6 × 21 + 15 × 35 = 7 + 126 + 525 = **658**. **[1]**

SECTION – E **(1 × 4 = 4)**

Case Study Based Question

Q23. **4 marks**



(i) Choose any 5 of the 12 members: ${}^{12}C_5 = (12 \times 11 \times 10 \times 9 \times 8)/(5 \times 4 \times 3 \times 2 \times 1) =$ **792** ways. **[1]**

(ii) Exactly 3 boys means 3 boys and 2 girls: ${}^{7}C_3 \times {}^{5}C_2 = 35 \times 10 =$ **350** ways. **[1]**

(iii)(a) At least 4 boys: (4 boys, 1 girl) or (5 boys, 0 girl). **[1]**

${}^{7}C_4 \times {}^{5}C_1 + {}^{7}C_5 \times {}^{5}C_0 = 35 \times 5 + 21 \times 1 = 175 + 21 =$ **196** ways. **[1]**

OR (b) Teams that contain both Aman and Diya: the other 3 members come from the remaining 10 students, in ${}^{10}C_3 = 120$ ways. **[1]**

Required number = total – teams with both = 792 – 120 = **672** ways. **[1]**

— End of Solutions —