CLASS XI – MATHEMATICS (041)

CHAPTER 7 : BINOMIAL THEOREM

PRACTICE PAPER – STEP-BY-STEP SOLUTIONS & MARKING SCHEME

Section A – Answer Key (1 mark each)

Q1	Q2	Q3	Q4	Q5	Q6	Q7
A	C	B	D	A	C	B
Q8	**Q9**	**Q10**	**Q11**	**Q12**	**Q13**	**Q14**
D	A	C	D	B	D	C

SECTION – A (14 × 1 = 14)

Multiple Choice and Assertion–Reason Questions

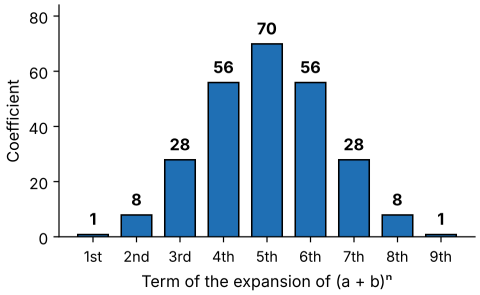
Q1. Answer: (A) — 1 mark

Expansion of $(a + b)^6$

Term no.	Term
1	a^6
2	$6a^5b$
3	?
4	$20a^3b^3$
5	$15a^2b^4$
6	$6ab^5$
7	b^6

The third term of $(a + b)^6$ is ${}^6C_2\, a^4b^2 = 15a^4b^2$.
(Check with the figure: the 4th term $20a^3b^3 = {}^6C_3\, a^3b^3$, and the coefficients 1, 6, 15, 20, 15, 6, 1 are symmetric.)

Q2. Answer: (C) — 1 mark



Adding the heights: 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = **256**.
Check: there are 9 terms, so n = 8, and the sum of coefficients is $2^8 = 256$.

Q3. Answer: (B) — 1 mark

$(a + b)^n$ has n + 1 terms. For n = 9 the number of terms is 9 + 1 = **10**.

Q4. Answer: (D) — 1 mark

$(1 + 2x)^5 = {}^5C_0 + {}^5C_1(2x) + {}^5C_2(2x)^2 + \ldots$ The x^2 term is ${}^5C_2 \times 4x^2 = 10 \times 4x^2 = 40x^2$.
Coefficient = **40**.

Q5. Answer: (A) — 1 mark

$(1 - x)^n = {}^nC_0 - {}^nC_1\, x + {}^nC_2\, x^2 - \ldots + (-1)^n\, {}^nC_n\, x^n$.
Put n = 7 and x = 1: the given sum $= (1 - 1)^7 =$ **0**.

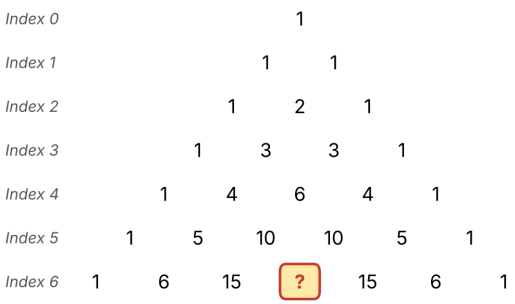
Q6. Answer: (C) — 1 mark

$(x + 1)^4 = x^4 + 4x^3 + 6x^2 + 4x + 1$ and $(x - 1)^4 = x^4 - 4x^3 + 6x^2 - 4x + 1$.
Subtracting: $8x^3 + 8x$. Answer: **$8x^3 + 8x$**.

Q7. Answer: (B) — 1 mark

$(101)^3 = (100 + 1)^3 = 100^3 + 3(100)^2(1) + 3(100)(1)^2 + 1^3 = 1000000 + 30000 + 300 + 1 =$ **1030301**.

Q8. Answer: (D) **1 mark**



Each number in Pascal's triangle is the sum of the two numbers just above it. In row 5 the two numbers above the hidden entry are 10 and 10.
Hidden number = 10 + 10 = **20**. (Also ${}^{6}C_{3} = 20$.)

Q9. Answer: (A) **1 mark**

$11^n = (1 + 10)^n = 1 + 10n + 10^2({}^{n}C_{2} + 10 \times {}^{n}C_{3} + \ldots)$.
So $11^n - 10n - 1 = 100 \times$ (a whole number), i.e. it is divisible by **100**.
(For n = 2: 121 – 21 = 100, which is not divisible by 1000 or 121, so the other options fail.)

Q10. Answer: (C) **1 mark**

The sum of the coefficients of a polynomial in x is its value at x = 1.
Sum = $(3 \times 1 - 2)^4 = 1^4 =$ **1**.

Q11. Answer: (D) **1 mark**

$(1.1)^{10} = (1 + 0.1)^{10} = 1 + 10(0.1) +$ other positive terms $= 2 +$ other positive terms.
Hence $(1.1)^{10}$ is **greater than 2**. (Its value is about 2.59.)

Q12. Answer: (B) **1 mark**

In $(x + y)^6 + (x - y)^6$ the terms having odd powers of y cancel and the even-power terms double:
$= 2[x^6 + 15x^4y^2 + 15x^2y^4 + y^6]$, which has **4** terms.

Q13. Answer: (D) **1 mark**

$(a + b)^n$ has n + 1 terms, not n. Assertion is false.
In each term the powers of a and b add up to n (e.g. $a^{n-r}b^r$). Reason is true → option (D).

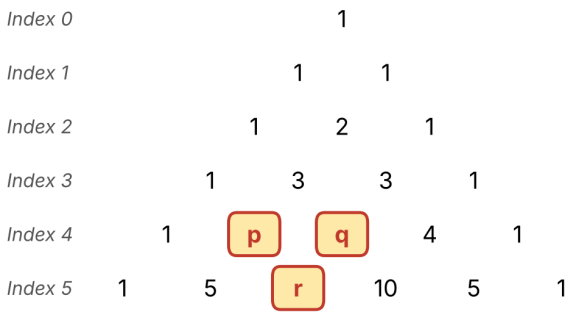
Q14. Answer: (C) **1 mark**

Putting x = 1 in $(1 + x)^8$ gives the sum of the coefficients = $2^8 = 256$. Assertion is true.
In general ${}^{n}C_{0} + {}^{n}C_{1} + \ldots + {}^{n}C_{n} = 2^n$, not $2^n - 1$. Reason is false → option (C).

SECTION – B (4 × 2 = 8)

Very Short Answer Questions

Q15. 2 marks



(i) Row 3 is 1, 3, 3, 1. Each entry of row 4 is the sum of the two entries above it: p = 1 + 3 = **4**, q = 3 + 3 = **6**. **[½]**

Row 4 is 1, 4, 6, 4, 1, so r = 6 + 4 = **10**. **[½]**

(ii) The row of index 5 is 1, 5, 10, 10, 5, 1. Taking a = 1, b = –x: **[½]**

$(1 - x)^5 = 1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5$. **[½]**

Q16. 2 marks

Using the row 1, 4, 6, 4, 1: $(x - 2/x)^4 = x^4 - 4x^3(2/x) + 6x^2(2/x)^2 - 4x(2/x)^3 + (2/x)^4$. **[1]**

$= x^4 - 8x^2 + 24 - 32/x^2 + 16/x^4$. **[1]**

Q17. 2 marks

(A) $(99)^4 = (100 - 1)^4 = 100^4 - 4(100)^3 + 6(100)^2 - 4(100) + 1$. **[1]**

$= 100000000 - 4000000 + 60000 - 400 + 1 =$ **96059601**. **[1]**

OR (B) $(1.03)^{200} = (1 + 0.03)^{200} = 1 + 200(0.03) +$ other positive terms. **[1]**

$= 1 + 6 +$ other positive terms $= 7 +$ other positive terms > 7. Hence $(1.03)^{200} > 7$. **[1]**

Q18. 2 marks

(A) $(a + b)^4 - (a - b)^4 = 2[{}^4C_1\, a^3b + {}^4C_3\, ab^3] = 8a^3b + 8ab^3$. **[1]**

Put $a = \sqrt{5}$, $b = 1$: $8(5\sqrt{5}) + 8\sqrt{5} = 40\sqrt{5} + 8\sqrt{5} =$ **$48\sqrt{5}$**. **[1]**

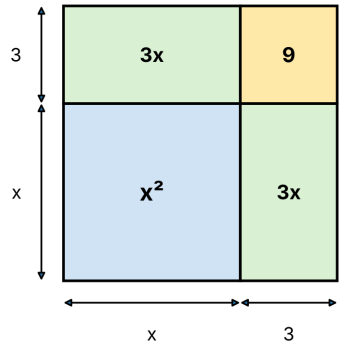
OR (B) The sum of all the binomial coefficients is $2^n = 512 = 2^9$, so **$n = 9$**. **[1]**

Number of terms $= n + 1 =$ **10**. **[1]**

SECTION – C (3 × 3 = 9)

Short Answer Questions

Q19. 3 marks



(i) The parts have areas $x \times x = x^2$, $x \times 3 = 3x$, $3 \times x = 3x$ and $3 \times 3 = 9$. Total area $= x^2 + 3x + 3x + 9$ $= \mathbf{x^2 + 6x + 9}$. **[1]**

(ii) $(x + 3)^2 = {}^2C_0\, x^2 + {}^2C_1\, x(3) + {}^2C_2\, (3)^2 = x^2 + 6x + 9$, which agrees with part (i). **[1]**

(iii) $(103)^3 = (100 + 3)^3 = 100^3 + 3(100)^2(3) + 3(100)(3)^2 + 3^3 = 1000000 + 90000 + 2700 + 27 =$ **1092727**. **[1]**

Q20. 3 marks

(A) $(3x - 1/x)^5$: use the row 1, 5, 10, 10, 5, 1 with $a = 3x$, $b = -1/x$. **[½]**

$= (3x)^5 - 5(3x)^4(1/x) + 10(3x)^3(1/x)^2 - 10(3x)^2(1/x)^3 + 5(3x)(1/x)^4 - (1/x)^5$ **[1½]**

$= 243x^5 - 405x^3 + 270x - 90/x + 15/x^3 - 1/x^5$. **[1]**

OR (B) $(102)^4 = (100 + 2)^4 = 100^4 + 4(100)^3(2) + 6(100)^2(2)^2 + 4(100)(2)^3 + 2^4$. **[1½]**

$= 100000000 + 8000000 + 240000 + 3200 + 16$ **[1]**

$=$ **108243216**. **[½]**

Q21. 3 marks

(A) Write $7^n = (1 + 6)^n = {}^nC_0 + {}^nC_1 \times 6 + {}^nC_2 \times 6^2 + \ldots + {}^nC_n \times 6^n$. **[1]**

$= 1 + 6n + 6^2({}^nC_2 + 6 \times {}^nC_3 + \ldots + 6^{n-2})$. **[1]**

So $7^n - 6n - 1 = 36k$, where $k = {}^nC_2 + 6 \times {}^nC_3 + \ldots + 6^{n-2}$ is a natural number. Hence it is divisible by 36. **[1]**

OR (B) $(x + y)^4 + (x - y)^4 = 2[x^4 + 6x^2y^2 + y^4] = 2x^4 + 12x^2y^2 + 2y^4$. **[1½]**

Put $x = \sqrt{3}$, $y = 1$: $2(9) + 12(3)(1) + 2 = 18 + 36 + 2 =$ **56**. **[1½]**

SECTION – D **(1 × 5 = 5)**

Long Answer Question

Q22. **5 marks**

(A) $(1 + 2x)^5 = 1 + 5(2x) + 10(2x)^2 + 10(2x)^3 + 5(2x)^4 + (2x)^5 = 1 + 10x + 40x^2 + 80x^3 + 80x^4 + 32x^5$. **[1]**

$(1 – 2x)^5 = 1 – 10x + 40x^2 – 80x^3 + 80x^4 – 32x^5$. **[1]**

Adding: $(1 + 2x)^5 + (1 – 2x)^5 = 2 + 80x^2 + 160x^4$ (the odd-power terms cancel). **[1]**

Put $x = 0.01$: then $1 + 2x = 1.02$ and $1 – 2x = 0.98$. **[1]**

$(1.02)^5 + (0.98)^5 = 2 + 80(0.0001) + 160(0.00000001) = 2 + 0.008 + 0.0000016 =$ **2.0080016**. **[1]**

OR (B) (i) $(1 + x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \ldots + {}^nC_n\, x^n$. Putting $x = 2$: **[1]**

${}^nC_0 + 2 \times {}^nC_1 + 2^2 \times {}^nC_2 + \ldots + 2^n \times {}^nC_n = (1 + 2)^n = 3^n$. **[1]**

(ii) Take $n = 5$: ${}^5C_0 + 2 \times {}^5C_1 + 4 \times {}^5C_2 + 8 \times {}^5C_3 + 16 \times {}^5C_4 + 32 \times {}^5C_5 = 3^5 =$ **243**. **[1]**

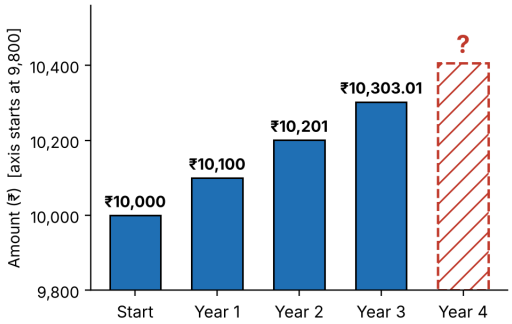
(iii) Put $x = –2$ in the same expansion: $(1 – 2)^n = {}^nC_0 – 2 \times {}^nC_1 + 2^2 \times {}^nC_2 – \ldots + (–2)^n \times {}^nC_n$. **[1]**

Hence ${}^nC_0 – 2 \times {}^nC_1 + 2^2 \times {}^nC_2 – \ldots + (–2)^n \times {}^nC_n = (–1)^n$. **[1]**

SECTION – E **(1 × 4 = 4)**

Case Study Based Question

Q23. **4 marks**



(i) $(1 + 0.01)^3 = 1 + 3(0.01) + 3(0.01)^2 + (0.01)^3 = 1 + 0.03 + 0.0003 + 0.000001 = 1.030301$. **[½]**
Amount = 10000 × 1.030301 = ₹10,303.01, which matches the bar for Year 3. **[½]**
(ii) $(1.01)^4 = 1 + 4(0.01) + 6(0.01)^2 + 4(0.01)^3 + (0.01)^4 = 1 + 0.04 + 0.0006 + 0.000004 + 0.00000001 = 1.04060401$. **[½]**
Amount at the end of Year 4 = 10000 × 1.04060401 = **₹10,406.04** (to the nearest paisa). **[½]**
(iii)(a) $(1.01)^{100} = (1 + 0.01)^{100} = 1 + 100(0.01) +$ other positive terms. **[1]**
$= 2 +$ other positive terms > 2. So the money more than doubles in 100 years. **[1]**
OR (b) $(0.99)^5 = (1 - 0.01)^5 \approx 1 - 5(0.01) + 10(0.01)^2$. **[1]**
$= 1 - 0.05 + 0.001 =$ **0.951**. **[1]**

— End of Solutions —