CLASS XI – MATHEMATICS (041)

CHAPTER 8 : SEQUENCES AND SERIES

PRACTICE PAPER – STEP-BY-STEP SOLUTIONS & MARKING SCHEME

Section A – Answer Key (1 mark each)

Q1	Q2	Q3	Q4	Q5	Q6	Q7
B	D	A	C	B	A	D
Q8	**Q9**	**Q10**	**Q11**	**Q12**	**Q13**	**Q14**
C	B	D	A	C	A	D

SECTION – A (14 × 1 = 14)

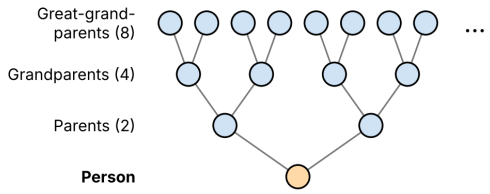
Multiple Choice and Assertion–Reason Questions

Q1. Answer: (B) — 1 mark



From the figure: 1 square needs 4 sticks, 2 squares need 7, 3 squares need 10. These numbers 4, 7, 10, . . . increase by 3 each time, so $a_n = 3n + 1$.
(Each extra square adds 3 sticks.) For n = 10: 3(10) + 1 = **31**.

Q2. Answer: (D) — 1 mark



The numbers of ancestors 2, 4, 8, . . . form a G.P. with a = 2, r = 2 and n = 6.
$S_6 = a(r^n - 1)/(r - 1) = 2(2^6 - 1)/(2 - 1) = 2 \times 63 =$ **126**.

Q3. Answer: (A) — 1 mark

$a_6 = (6 + 1)(6 - 2)(6 + 3) = 7 \times 4 \times 9 =$ **252**.

Q4. Answer: (C) — 1 mark

$a_1 = 2$; $a_2 = 3(2) - 1 = 5$; $a_3 = 3(5) - 1 = 14$; $a_4 = 3(14) - 1 =$ **41**.

Q5. Answer: (B) — 1 mark

a = 3, r = 6/3 = 2. $a_8 = ar^7 = 3 \times 2^7 = 3 \times 128 =$ **384**.

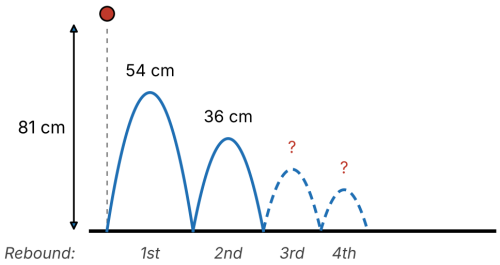
Q6. Answer: (A) — 1 mark

$a_2 = ar = 6$ and $a_5 = ar^4 = 162$. Dividing: $r^3 = 162/6 = 27$.
So r = **3**.

Q7. Answer: (D) — 1 mark

a = 2, r = 3, n = 6. $S_6 = 2(3^6 - 1)/(3 - 1) = 3^6 - 1 = 729 - 1 =$ **728**.

Q8. Answer: (C) — 1 mark



The rebound heights are 81 × 2/3 = 54, then 54 × 2/3 = 36, . . . : a G.P. with first term 54 and common ratio 2/3.
4th rebound = $54 \times (2/3)^3 = 54 \times 8/27 =$ **16 cm**. (The heights are 54, 36, 24, 16.)

Q9. Answer: (B) **1 mark**

5, G, 20 are in G.P., so G is the G.M. of 5 and 20: $G^2 = 5 \times 20 = 100$.
G is positive, so G = **10**.

Q10. Answer: (D) **1 mark**

The series is $2 + 2^2 + 2^3 + 2^4 = 2 + 4 + 8 + 16 =$ **30**.
(Check with the formula: a = 2, r = 2, n = 4 → $2(2^4 - 1)/(2 - 1) = 30$.)

Q11. Answer: (A) **1 mark**

$(a + b)/2 = 13 \Rightarrow a + b = 26$ and $\sqrt{(ab)} = 12 \Rightarrow ab = 144$.
$(a - b)^2 = (a + b)^2 - 4ab = 676 - 576 = 100 \Rightarrow a - b = \pm 10$.
Solving with $a + b = 26$: the numbers are **8 and 18**.

Q12. Answer: (C) **1 mark**

a = 1, r = 2: $S_n = (2^n - 1)/(2 - 1) = 255 \Rightarrow 2^n = 256 = 2^8$.
So n = **8**.

Q13. Answer: (A) **1 mark**

Using the rule: $a_1 = 1$, $a_2 = 1$, $a_3 = 2$, $a_4 = 3$, $a_5 = 5$, $a_6 = 8$, $a_7 = 5 + 8 = 13$. Assertion is true.
Reason states exactly the rule that generates the sequence, so it explains the assertion → option (A).

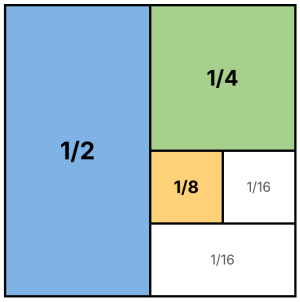
Q14. Answer: (D) **1 mark**

$A - G = (a + b)/2 - \sqrt{(ab)} = (\sqrt{a} - \sqrt{b})^2/2 \geq 0$, so $A \geq G$; the A.M. is never less than the G.M. The Assertion (A.M. less than G.M.) is false.
The Reason is the true calculation just shown → option (D).

SECTION – B (4 × 2 = 8)

Very Short Answer Questions

Q15. 2 marks



(i) Each term is half the previous one, so the common ratio is **1/2**. The next two terms are **1/16 and 1/32**. **[1]**

(ii) The first four terms 1/2, 1/4, 1/8, 1/16 form a G.P. with a = 1/2, r = 1/2, n = 4. **[½]**

$S_4 = (1/2)[1 - (1/2)^4]/(1 - 1/2) = 1 - 1/16 =$ **15/16** square unit. **[½]**

Q16. 2 marks

$a_1 = 2$; $a_2 = 2(2) - 1 = 3$; $a_3 = 2(3) - 1 = 5$; $a_4 = 2(5) - 1 = 9$; $a_5 = 2(9) - 1 = 17$. **[1]**

First five terms: **2, 3, 5, 9, 17**. Corresponding series: **2 + 3 + 5 + 9 + 17 + . . .** **[1]**

Q17. 2 marks

(A) a = 3, r = 2. $a_{10} = ar^9 = 3 \times 2^9 = 3 \times 512 =$ **1536**. **[1]**

$a_n = ar^{n-1} =$ **$3 \times 2^{n-1}$**. **[1]**

OR (B) a = 5, r = 2. Let 640 be the nth term: $5 \times 2^{n-1} = 640 \Rightarrow 2^{n-1} = 128 = 2^7$. **[1]**

So n – 1 = 7, i.e. n = 8. The **8th term** is 640. **[1]**

Q18. 2 marks

(A) a = 2, r = 3, n = 5: $S_5 = 2(3^5 - 1)/(3 - 1)$. **[1]**

$= 243 - 1 =$ **242**. **[1]**

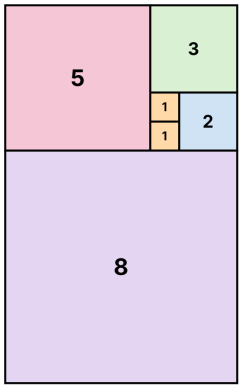
OR (B) $S_n = 3 + 33 + 333 + \ldots = (3/9)[9 + 99 + 999 + \ldots] = (1/3)[(10 - 1) + (10^2 - 1) + (10^3 - 1) + \ldots$ n terms]. **[1]**

$= (1/3)[(10 + 10^2 + \ldots + 10^n) - n] =$ **$(1/3)[10(10^n - 1)/9 - n]$**. **[1]**

SECTION – C (3 × 3 = 9)

Short Answer Questions

Q19. 3 marks



(i) Each side is the sum of the previous two: 5 + 8 = 13 and 8 + 13 = 21. The next two sides are **13 and 21**. **[1]**

(ii) $a_1 = a_2 = 1$ and $\mathbf{a_n = a_{n-2} + a_{n-1}}$ **for n > 2** (the Fibonacci recurrence). **[1]**

(iii) Continuing: $a_7 = 13$, $a_8 = 21$, $a_9 = 34$, $a_{10} = 55$. Side of the 10th square = **55**; a_{10}/a_9 = **55/34**. **[1]**

Q20. 3 marks

(A) $a_4 = ar^3 = 24$ and $a_7 = ar^6 = 192$. Dividing: $r^3 = 8 \Rightarrow r = 2$. **[1]**

Then $a(2)^3 = 24 \Rightarrow a = 3$. **[1]**

$a_{10} = ar^9 = 3 \times 2^9 =$ **1536**. **[1]**

OR (B) Let the terms be a/r, a, ar. Product: $(a/r)(a)(ar) = a^3 = 64 \Rightarrow a = 4$. **[1]**

Sum: $4/r + 4 + 4r = 14 \Rightarrow 4/r + 4r = 10 \Rightarrow 2r^2 - 5r + 2 = 0 \Rightarrow (2r - 1)(r - 2) = 0 \Rightarrow r = 2$ or $1/2$. **[1]**

For r = 2 the terms are **2, 4, 8**; for r = 1/2 they are **8, 4, 2**. **[1]**

Q21. 3 marks

(A) a = 2, r = 2: $S_n = 2(2^n - 1)/(2 - 1) = 510$. **[1]**

$2^n - 1 = 255 \Rightarrow 2^n = 256 = 2^8$. **[1]**

So **n = 8** terms are needed. **[1]**

OR (B) Let G_1, G_2, G_3 be the three numbers: 2, G_1, G_2, G_3, 162 is a G.P. with a = 2 and 162 as the 5th term. **[1]**

$162 = 2r^4 \Rightarrow r^4 = 81 \Rightarrow r = \pm 3$ (real values). **[1]**

For r = 3: **6, 18, 54**. For r = –3: **–6, 18, –54**. **[1]**

SECTION – D **(1 × 5 = 5)**

Long Answer Question

Q22. **5 marks**

(A)(i) $A = (a + b)/2$ and $G = \sqrt{(ab)}$. Then $A - G = (a + b - 2\sqrt{(ab)})/2$. **[1]**

$= (\sqrt{a} - \sqrt{b})^2/2 \geq 0$, since a square is never negative. Hence **A ≥ G**. **[1]**

(ii) $(a + b)/2 = 17 \Rightarrow a + b = 34$ and $\sqrt{(ab)} = 15 \Rightarrow ab = 225$. **[1]**

$(a - b)^2 = (a + b)^2 - 4ab = 1156 - 900 = 256 \Rightarrow a - b = \pm 16$. **[1]**

Solving $a + b = 34$ with $a - b = 16$ (or -16): the numbers are **25 and 9**. **[1]**

OR (B)(i) The numbers of letters in the sets are 3, 9, 27, . . . : a G.P. with $a = 3$, $r = 3$. In the nth set: 3^n letters; in the 6th set: $3^6 =$ **729**. **[1]**

(ii) Total in 6 sets $= S_6 = 3(3^6 - 1)/(3 - 1) = 3 \times 728/2$. **[1]**

= **1092** letters. **[1]**

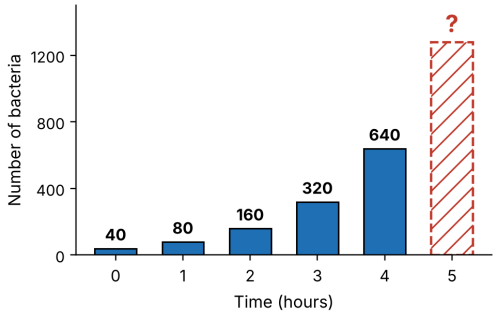
(iii) Postage = 1092 × ₹5 = **₹5460**. **[1]**

(iv) $S_n = 3(3^n - 1)/2 > 5000 \Rightarrow 3^n > 3334.3$. As $3^7 = 2187$ ($S_7 = 3279$) is too small and $3^8 = 6561$ ($S_8 = 9840$) works, the least number of sets is **8**. **[1]**

SECTION – E **(1 × 4 = 4)**

Case Study Based Question

Q23. **4 marks**



(i) The numbers 40, 80, 160, 320, 640 form a G.P. with common ratio 80/40 = **2**. Number at the end of the 5th hour = 640 × 2 = **1280**. **[1]**

(ii) The number at the end of the nth hour is $a_n = 40 \times 2^n$. For n = 8: 40 × 256 = **10240**. **[1]**

(iii)(a) We need $40 \times 2^n > 5000 \Rightarrow 2^n > 125$. **[1]**

Since $2^6 = 64$ and $2^7 = 128$, the least n is 7. The number first exceeds 5000 after **7 hours** (40 × 128 = 5120). **[1]**

OR (b) The numbers at the end of hours 1 to 6 are 80, 160, 320, . . . : a G.P. with a = 80, r = 2, n = 6. **[1]**

$S_6 = 80(2^6 - 1)/(2 - 1) = 80 \times 63 = \mathbf{5040}$. **[1]**

— End of Solutions —